NEW AMERICAN SERIES.

THE NEW AMERICAN ARITHMETIC

PART 3

PHILADELPHIA:
J. H. BUTLER & Co.

8. They are all N… …e present time.

From Rev. F. W. Holland

By

Samuel Mccutchen

and

George M. Sayre.

NEW AMERICAN SERIES.

THE NEW AMERICAN ARITHMETIC

PART 3

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PREFACE.

THE NEW AMERICAN ARITHMETIC, Part Third, is designed for the use of those who have completed Part Second of the series, or some similar work.

It begins, therefore, with *Denominate Fractions*, and thus reviews the closing topics in the preceding volume, Decimals and Denominate Numbers.

Percentage and its various applications, *Profit and Loss*, *Commission*, *Brokerage*, *Insurance*, *Interest*, *Stocks*, *Partial Payments*, *Banking*, and *Investments*, all of which are of special value in qualifying pupils for mercantile business, are treated fully and practically.

Ratio, *Proportion*, *Partnership*, *Equation of Payments*, and *Averaging of Accounts*, are grouped together, as having much in common in their mode of treatment.

A short chapter of problems in *Analysis* is given, and *Involution* and *Evolution* are next explained, preparatory to the article on *Mensuration*, in which much information of a practical character is embodied.

The subjects thus presented complete the topics of the greatest importance to pupils, and as much of the science of arithmetic as is usually taught in our common schools.

Such other subjects as are generally found in higher arithmetics are then treated, for the benefit of those who have time to devote to these branches.

S. M.
G. M. S.

PHILADELPHIA, 1877.

ELECTROTYPED BY MACKELLAR, SMITHS & JORDAN, PHILADELPHIA.
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THE

NEW AMERICAN ARITHMETIC.

PART III.

DENOMINATE FRACTIONS.

A **Denominate Fraction** is a fraction applied to a denomination; as, £$\frac{9}{12}$, $\frac{8}{18}$ gal., .015 of a mile.

Note.—In all examples where the word *fraction* is used, *common* fraction is meant.

CASE I.

To reduce a denominate fraction to an equivalent fraction of a lower denomination.

ORAL EXERCISES.

1. Reduce $\frac{3}{10}$ qt. to the fraction of a pint.

ANALYSIS.—In one quart there are 2 pints, and in $\frac{3}{10}$ qt. there are $\frac{3}{10}$ times 2 pints = $\frac{6}{10}$ or $\frac{3}{5}$ pt.

2. Reduce .3 qt. to the decimal of a pint.

ANALYSIS.—In one quart there are 2 pints, and in .3 qt. there are .3 times 2 pints = .6 pt.

3. What fraction of a peck is $\frac{3}{50}$ of a bushel?

4. What decimal of a peck is .06 of a bushel?

5. Reduce $\frac{1}{20}$ of a drachm to the fraction of a grain.

6. Reduce .05 of a drachm to the decimal of a grain.

7. What decimal of a day is .08 of a week?

8. $\frac{1}{24}$ *of a foot is* what fraction of an inch?

WRITTEN EXERCISES.

1. Reduce $\frac{1}{5000}$ of a ton to the fraction of a pound.

$\frac{1}{5000} \times \frac{20}{1} \times \frac{100}{1} = \frac{2}{5}$ lb.

ANALYSIS.—In this example we multiply, as in reduction descending, by the numbers required to reduce tons to pounds, applying the rule for the multiplication of common fractions.

2. Reduce .0002 T. to the decimal of a pound.

.0002
20
.0040
100
.4000 lb.

ANALYSIS.—We multiply by the same numbers in this as in the preceding example, applying the rule for the multiplication of decimals.

RULE.

Multiply the given fraction, as in reduction of denominate numbers, by the numbers required to reduce it to the proper denomination.

3. Reduce $\frac{1}{480}$ of a pound to the fraction of a penny.

4. .015 bu. is what decimal of a pint?

5. What fraction of a pint is $\frac{3}{32}$ of a gallon?

6. Reduce $\frac{1}{363}$ sq. rd. to the fraction of a sq. ft.

7. Reduce .0005 hr. to the decimal of a second.

8. $\frac{1}{396}$ of a rod is what fraction of an inch?

9. $\frac{7}{4800}$ ℥ is what fraction of a grain?

10. What decimal of a sq. ft. is .035 sq. yd.?

11. Reduce $\frac{1}{3700}$ of a degree to the fraction of a second.

12. What decimal of a cubic foot is $\frac{1}{1280}$ of a cord?

13. Change $\frac{1}{57600}$ of a pound Troy to the decimal of a grain.

14. Change $\frac{11}{250000}$ of a ton to the fraction of an ounce.

15. Reduce $\frac{25}{86600}$ of a leap year to the fraction of an hour.

CASE II.

To reduce a denominate fraction to an equivalent fraction of a higher denomination.

ORAL EXERCISES.

1. What part of a foot is $\frac{3}{5}$ of an inch?

ANALYSIS.—One inch is $\frac{1}{12}$ of a foot, and $\frac{3}{5}$ of an inch is $\frac{3}{5}$ of $\frac{1}{12} = \frac{1}{20}$ of a foot; or $\frac{3}{5} \div 12 = \frac{1}{20}$ of a foot.

2. What part of a foot is .6 of an inch?

ANALYSIS.—$.6 \div 12 = .05$ of a foot.

3. What fraction of a pound is $\frac{5}{6}$ of a shilling?

4. What decimal of a gallon is .24 qt.?

5. What fraction of a week is $\frac{7}{12}$ of a day?

6. $\frac{2}{3}$ of a foot is what fraction of a yard?

7. $\frac{9}{10}$ of a dime is what fraction of a dollar?

8. .9 of a dime is what decimal of a dollar?

9. Half a peck of oats is what fraction of a bushel?

10. What fraction of an eagle is a quarter of a dollar?

WRITTEN EXERCISES.

1. $\frac{2}{5}$ of a lb. is what fraction of a ton?

$$\frac{2}{5} \div \frac{100}{1} \div \frac{20}{1} = \frac{2}{5} \times \frac{1}{100} \times \frac{1}{20} = \frac{1}{5000} \text{ T.}$$

ANALYSIS.—In this example we divide, as in reduction ascending, by the numbers required to reduce pounds to tons, applying the rule for the division of common fractions.

2. .4 of a lb. is what decimal of a ton?

1|00).40|00
2|0).004|0
.0002 tons.

ANALYSIS.—We divide by the same numbers in this as in the preceding example, applying the rule for the division of decimals.

RULE.

Divide the given fraction, as in reduction of denominate numbers, by the numbers required to reduce it to the proper denomination.

3. $\frac{10}{12}$ of a penny is what fraction of a pound?

4. Reduce .5 of a pint to the decimal of a gallon.

5. Reduce $\frac{11}{20}$ of an inch to the fraction of a rod.

6. Reduce 36 half inches to the fraction of a yard.

7. .480 of a sq. rod is what decimal of an acre?

8. $\frac{2700}{3}$ of a second is what fraction of an hour?

9. $\frac{1}{10}$ of a cent is what part of an eagle?

10. What part of an eagle is .1 of a cent?

CASE III.

To find the value of a denominate fraction in whole numbers of a lower denomination.

ORAL EXERCISES.

1. Reduce $\frac{1}{2}$ of a bushel to pecks.

ANALYSIS.—In one bushel there are 4 pecks, and in $\frac{1}{2}$ of a bushel there are $\frac{1}{2}$ of 4 pecks = 2 pecks.

2. Reduce .5 of a bushel to pecks.

ANALYSIS.—$.5 \times 4 = 2.0$ pecks.

3. How many inches in $\frac{1}{3}$ of a yard?

4. How many cents in $\frac{4}{5}$ of a dollar?

5. In .8 of a dollar there are how many cents?

6. How many seconds in $\frac{5}{12}$ of a minute?

7. In .01 of a cwt. there are how many pounds?

8. Reduce $\frac{4}{5}$ of a pound to shillings.

9. How many glasses, each containing 3 gills, can be filled from $\frac{3}{4}$ of a gallon of water?

WRITTEN EXERCISES.

1. Reduce $\frac{3}{8}$ of a gallon to quarts and pints.

$\frac{3}{8} \times 4 = \frac{12}{8} = 1\frac{1}{2}$ qt.
$\frac{1}{2} \times 2 = \frac{2}{2} = 1$ pt.
Ans. 1 qt. 1 pt.

ANALYSIS.—We reduce gallons to quarts by multiplying by 4. $\frac{3}{8} \times 4 = \frac{12}{8}$ qt. $= 1\frac{1}{2}$ qt. We then reduce the $\frac{1}{2}$ qt. to pints, by multiplying by 2. $\frac{1}{2} \times 2 = 1$ pint; and the entire result is 1 qt. 1 pt.

2. Reduce .375 of a gallon to quarts and pints.

.375
4
1|.500
2
1.000

ANALYSIS.—We multiply the gallons by 4, as in the preceding operation. $.375 \times 4 = 1.500$ qt. We then reduce the .500 qt. to pints, by multiplying by 2. $.500 \times 2 = 1.000$ pt.; and the entire result is 1 qt. 1 pt.

Ans. 1 qt. 1 pt.

RULE.

Multiply the given fraction by the number required to reduce it to the next lower denomination. When the product is greater than a unit, change to a whole or mixed number, multiply the fractional part, if any, by the number required to reduce it to the next lower denomination, and proceed as before, until the reduction is completed.

3. Reduce $\frac{8}{9}$ of a ton to pounds and ounces.

4. What is the value of $\frac{4}{5}$ of a rod in yards, feet, and inches?

5. Reduce .8534 bu. to pecks, quarts, and pints.

6. Add $\frac{5}{7}$ of a day and $\frac{4}{7}$ of an hour, and give the result in hours, minutes, and seconds.

7. From £.3805 subtract .3805 s., giving the result in shillings, pence, and farthings.

8. Three farms contain respectively $120\frac{3}{8}$ A., $160\frac{4}{5}$ A., and $185\frac{9}{16}$ A.; how many acres and square rods are there in all?

9. If I buy $12\frac{4}{5}$ cwt. of flour from one man, 16.25 cwt. from another, and $18\frac{3}{4}$ cwt. from a third, and then sell 19.75 cwt., how many cwt. and lb. do I have left?

10. Find the sum of $\frac{5}{12}$ of a day, $\frac{3}{4}$ of an hour, $\frac{5}{8}$ of an hour, and $\frac{11}{16}$ of a minute.

11. From $\frac{3}{8}$ of an oz. of gold I sold $\frac{7}{8}$ of a pwt.; how much had I remaining?

12. Find the sum of 18.546°, 15.314°, and 1.09°, in degrees, minutes, and seconds.

13. Reduce $\frac{128}{16}$ pints to the fraction of a bushel, add $\frac{7}{8}$ of a bushel to the result, and give the answer in bushels.

14. Reduce $\frac{7}{8}$ of a bushel to the fraction of a pint, add $\frac{128}{16}$ pt., and change the result to bushels.

CASE IV.

To reduce one denominate number to the fractional part of another.

ORAL EXERCISES.

1. What fraction of an hour is 15 minutes?

ANALYSIS.—In one hour there are 60 minutes, and 15 minutes $= \frac{15}{60}$ or $\frac{1}{4}$ of an hour.

2. What decimal of an hour is 15 minutes?

ANALYSIS.—15 minutes $= \frac{1}{4}$ of an hour. $\frac{1}{4}$ of an hour $= .25$ of an hour.

3. What fraction of a gallon is 3 quarts? What decimal of a gallon?

4. Reduce 50 cents to the decimal of a dollar.

5. What fraction of a bushel is 16 qt.? What decimal of a bushel?

6. What fraction of a pound is 15 s.? What decimal of a pound?

7. Reduce 1 ft. 6 in. to the fraction of a yard. To the decimal of a yard.

8. What fraction of 3 feet is 18 inches?

9. Reduce 30 minutes to the decimal of 2 hours.

10. What fraction of 3 bushels is 3 pecks?

WRITTEN EXERCISES.

1. Reduce 10 oz. 18 pwt. 12 gr. to the fraction of a pound.

10 oz. 18 pwt. 12 gr. = 5244 gr.
1 lb. = 5760 gr.
$\frac{5244}{5760} = \frac{437}{480}$.

ANALYSIS.—In 10 oz. 18 pwt. 12 gr. there are 5244 gr., and in 1 lb. there are 5760 gr. 5244 gr. = $\frac{5244}{5760}$ or $\frac{437}{480}$ of a pound.

2. Reduce 10 oz. 18 pwt. 12 gr. to the decimal of a pound.

10 oz. 18 pwt. 12 gr. = 5244 gr.
1 lb. = 5760 gr.
$\frac{5244}{5760} = \frac{437}{480} = .9104+$

ANALYSIS.—10 oz. 18 pwt. 12 gr. = $\frac{437}{480}$ of a pound. $\frac{437}{480}$ of a pound reduced to a decimal = .9104 + of a pound.

RULE.

Reduce the given numbers to the lowest denomination mentioned in either. Write the number which is to become the fractional part, for the numerator of the required fraction, and the remaining number for the denominator, and change the fraction to its lowest terms, or to a decimal if necessary.

3. Reduce 16 s. 8 d. 1 far. to the fraction of a pound. To the decimal of a pound.

4. Reduce 12 cwt. 90 lb. to the decimal of a ton.

5. From a farm containing 120 A. 60 sq. rd., there were sold 40 A. 29 sq. rd.; what fractional part of the farm was sold?

6. If £1 is worth $4.8665, what is the value of £5 9 s. 6 d.?

7. Reduce 3 pk. 7 qt. 1 pt. to the decimal of a bushel.

8. Reduce 7 oz. 5 pwt. 20 gr. to the decimal of a pound.

9. What fraction of a cord is 65 cu. ft. 144 cu. in.?

10. What decimal of a week is 1 da. 6 hr. 30 min.?

PERCENTAGE.

Per Cent. is an abbreviation of the Latin words *per centum*, meaning by the hundred, and any per cent. of a number is that many *hundredths* of the number.

ILLUSTRATION.—In a town of 600 people, 3 per cent. can neither read nor write; this means that three in every hundred, or eighteen in all, can neither read nor write.

In the above example we find *three* points to be considered: the *base* (600 people), the *rate* (3 per cent.), and the *percentage* (18 people).

The **Base** is the number on which the percentage is computed.

The **Rate** or **Rate Per Cent.** is the number in each hundred or the number of hundredths to be taken.

The **Percentage** is the result obtained by taking as many hundredths of the base as are indicated by the rate.

The **sign** % is generally used instead of the words *per cent.;* thus, 6%, 8½%.

As the rate per cent. is a number of hundredths, it may be expressed either decimally, or as a common fraction. Thus:—

1% = .01 or $\frac{1}{100}$.	2½% = .02½ or $\frac{2\frac{1}{2}}{100}$.
3% = .03 or $\frac{3}{100}$.	33⅓% = .33⅓ or $\frac{33\frac{1}{3}}{100}$.
12% = .12 or $\frac{12}{100}$.	½% = .00½ or $\frac{\frac{1}{2}}{100}$.
100% = 1.00 or $\frac{100}{100}$.	¼% = .00¼ or $\frac{\frac{1}{4}}{100}$.
115% = 1.15 or $\frac{115}{100}$.	⅛% = .00⅛ or $\frac{\frac{1}{8}}{100}$.

Note.—In expressing any rate per cent., when the decimal point is used, the words per cent. and the sign % are omitted; when either the sign or the words are used, the decimal point is omitted.

CASE I.

To find any per cent. of a number.

ORAL EXERCISES.

1. Express 5% as a common fraction in its lowest terms.

ANALYSIS.—$5\% = \frac{5}{100} = \frac{1}{20}$.

2. Express in the same manner 10%, 20%, 25%, 40%, 50%, 60%, 75%, 80%, 12½%, 16⅔%, 33⅓%, 66⅔%.

3. What is 10% of $80?

ANALYSIS.—$10\% = \frac{10}{100}$ or $\frac{1}{10}$. $\frac{1}{10}$ of $80 = $8.

4. What is 20% of 60? Of 80? Of 100? Of ¼?

5. What is 50% of 12 horses? Of 60 dollars?

6. What is $33\frac{1}{3}\%$ of 18? Of 24? Of 90?

7. I bought two dozen hams, of which 25% were unfit for use; how many were good?

8. A farmer raised 500 bushels of corn, and sold 20% of it; how many bushels did he sell?

9. A man received $12\frac{1}{2}\%$ of $800; how many dollars did he receive?

10. From a hhd. containing 63 gallons, $33\frac{1}{3}\%$ leaked out; how much remained?

11. In a school of boys and girls containing 200 pupils, 50% are girls; how many are boys?

12. A received 20% of 100 dollars, and B 75%; how much did each get?

13. A man buys a sewing-machine and pays 10% of the price every week; in how many weeks will he have paid for it?

WRITTEN EXERCISES.

1. What is 8% of 720 gallons?

720
.08
57.60 gal.

ANALYSIS.—We express the rate % decimally and multiply; 8% = .08, and .08 of 720 gal. = 57.6 gal.

RULE.

Multiply the given number, or base, by the rate expressed decimally; the product will be the percentage.

What is:—

2. 8% of 1220 men?
3. $3\frac{3}{4}\%$ of $1000?
4. 65% of 11692?
5. $33\frac{1}{3}\%$ of 1200 sheep?
6. $\frac{1}{2}\%$ of 960?
7. 17% of $165.25?
8. 36% of $2000?
9. $\frac{5}{8}\%$ of 1600 pounds?
10. 112% of 540 yd.?
11. 99% of 99?

12. A horse costing $150 was sold at a loss of 20%; what was the loss?

13. On a bill of goods amounting to $183.76, 5% was deducted for cash; how much was paid?

14. A clerk receives a salary of $1800; he pays $16\frac{2}{3}\%$ of this for rent, $66\frac{2}{3}\%$ for family expenses, and saves the remainder; how much does he save?

15. An agent collects $1817.12, of which he retains $2\frac{1}{2}\%$; how much does he hand over to his employer?

16. A man bought a house for $6500, and sold it for 6% more than he gave for it; how much did he get for the house?

17. A merchant, failing in business, paid his creditors 80 cents on the dollar; what % did he pay, and how much did the man receive to whom he owed $1854.96?

18. What is 5% of £18 12 s. 6 d.?

CASE II.

To find what per cent. one number is of another.

ORAL EXERCISES.

1. What % of 80 is 20?

ANALYSIS.—20 is $\frac{1}{4}$ of 80. $\frac{1}{4}=.25$, or 25%.

2. What % of 20 is 10? Of 40 is 8? Of 60 is 20?

3. What % of 25 is $12\frac{1}{2}$? Of 50 is $16\frac{2}{3}$? Of 1 is $\frac{1}{4}$?

4. 7 is what % of 14? Of 21? Of 35? Of 70?

5. 90 is what % of 100? Of 180? Of 90?

6. What % of $\frac{1}{2}$ is $\frac{1}{4}$? Of $12\frac{1}{2}$ is $6\frac{1}{4}$? Of $\frac{1}{3}$ is $\frac{1}{15}$?

7. A boy having 12 cents gave away 3; what % did he give away?

8. A boy having 12 cents gave away 3; what % did he keep?

9. A horse costing \$120 was sold for \$60; what per cent. of the cost did he sell for?

10. A man having \$50 paid \$10 for a coat; what per cent. of his money did the coat cost?

11. A farmer lost by decay 10 bushels of potatoes out of 40 bushels; what % did he lose?

12. Louis has half a dollar and John has 25 cents; what % of Louis's money is John's?

13. A man weighing 150 pounds lost 30 pounds through sickness; what % did he lose?

14. I have 10 cents, with which I buy a book, and sell it again for 15 cents; what % do I gain on my money?

15. I buy a watch for \$10, and sell it for \$15; what % do I gain on the money which I lay out?

WRITTEN EXERCISES.

1. A man owing \$750 paid \$624; what per cent did he pay?

$$
\begin{array}{r}
750)624.00(.83\tfrac{1}{3} \text{ or } 83\tfrac{1}{3}\% \\
6000 \\
\hline
2400 \\
2250 \\
\hline
150 \\
\hline
750
\end{array}
= \frac{1}{5}
$$

PROOF.

\$750
$.83\frac{1}{3}$
2250
6000
150
\$624.00

ANALYSIS.—624 is $\frac{624}{750}$ of 750. $\frac{624}{750}$ changed to a decimal and carried to the *hundredths* place (per cent. being so many hundredths) = $.83\frac{1}{3}$ or $83\frac{1}{3}\%$.

RULE.

Divide the percentage by the base, extending the division to hundredths.

2. What % of 1224 is 816?

3. What % of $17\frac{3}{4}$ is $6\frac{5}{8}$?

4. A clerk whose salary is $1500 pays $300 for rent; what % is that of his salary?

5. A city whose population was 45000 in 1860, had increased to 60000 in 1870; what was the increase, and what % is it of the population in 1860?

6. In a district school of 46 pupils, 12 are studying history; what % is that of the whole number?

7. A man sold a watch for $175, that had cost him $150; what did he gain, and what % of the original cost is the gain?

8. $\frac{4}{5}$ of a barrel of flour was damaged by water; what % remained sound?

9. What % of 3 gal. 2 qt. 1 pt. is 1 gal. 3 qt.?

10. A London merchant owing £198 9s. paid £66 3s.; what % did he pay?

11. What % of $\frac{3}{4}$ is $\frac{8}{12}$? $\frac{9}{28}$ is what % of $\frac{3}{7}$?

12. A farmer's flock of sheep decreased from 480 to 320; what % did he lose?

13. What % of the weight of a barrel of pork is the weight of a barrel of flour?

14. A merchant fails in business, owing $25000; what % can he pay, if his property is worth 8333.33\frac{1}{3}$?

15. If the standard for gold coin in the United States is 9 parts of gold to 1 part of alloy, what is the % of alloy in an eagle?

16. A farmer feeds his horses a mixture of oats and corn, in the proportion of 7 qt. of oats to 1 qt. of corn; what is the per cent. of oats in a peck of the mixture? In a bushel? In 100 bushels?

17. What per cent. of 3.1416 is .7854?

CASE III.

To find a number when any per cent. of it is given.

ORAL EXERCISES.

1. 12 is 3% of what number?

ANALYSIS.—If 12 is 3% of some number, 1% of that number is $\frac{1}{3}$ of 12 = 4, and 100%, or the number, is 100 times 4 = 400.

2. 20 is 10% of what number? 30 is 5% of what number?

3. 16 is 8% of what number? 4 is 20% of what number?

4. 25 is 25% of what number? 50% of what number?

5. Charles has 12 dollars, which is 6% of what Robert has; how much has Robert?

6. A man buying a house made a payment of $400, which was 10% of its price; what was the price?

7. 80% of the pupils of a school were dismissed, while the remainder of the pupils, 40 in number, were detained; what was the entire number?

ANALYSIS.—100% or the entire number − 80% = 20% detained. If 20% = 40 pupils, etc.

8. A man sold a horse for 20% more than he gave for him, receiving $240; what did he give for the horse?

ANALYSIS.—100% or the cost of the horse + 20% gain = 120% = $240, etc.

9. A man sold a horse for 20% less than he gave for him, receiving $160; what did he give for the horse?

ANALYSIS.—100% − 20% = 80% = $160, etc.

10. 250 is 25% more than what number?

11. 250 is 25% less than what number?

WRITTEN EXERCISES.

1. $246 is 3% of what sum?

	PROOF.
.03)$246.00	$8200
$8200	.03
	$246.00

ANALYSIS.—In this example we have the percentage and rate given, to find the base; now as the base multiplied by the rate gives the percentage, it is evident that the percentage divided by the rate must give the base.

RULE.

Divide the percentage by the rate expressed as a decimal.

2. A man's taxes, at the rate of $2\frac{1}{2}$%, amount to $150; what is the valuation of his property?

3. A merchant failing in business pays 45 cents on the dollar; how much did he owe the man to whom he paid $1125?

4. An army lost 15% of its number in battle, 4% by desertion, and 11% by sickness, leaving but 14000 men fit for duty; what was the original number?

5. A man drew from bank $118.65, which was 15% of what he had deposited; what amount had he at first?

6. A man drew from bank $118.65, which was 15% of what he had left; what amount had he at first?

7. I expend 60% of my income, and save the balance, which is $1200; what is my income?

8. A farmer sold 450 bushels of corn, which was $33\frac{1}{3}$% of his entire crop; how much had he left?

9. A clerk's salary was increased 8%, making it $2700; what was his original salary?

10. $375 is 75% of the cost of my horse and carriage; what did they cost?

11. A workman's wages were decreased 10%, leaving him but $1.35 per day; what did he receive before the reduction?

12. A man lost 20% of his weight through sickness, and then weighed 120 pounds; what was his original weight?

13. A merchant sold a barrel of flour for $9, which was 12½% more than it cost him; what did it cost him?

14. What number increased by 33⅓% of itself will equal 888?

REVIEW PROBLEMS.

ORAL EXERCISES.

1. What is 20% of 40? 40 is 20% of what number? What % of 40 is 20?

2. What % of 20 is 5? What is 5% of 20? 20 is 5% of what number?

3. What % of 5 cents is 1 cent? Of 50 cents is 10 cents?

4. What % of 10 barrels is 2 barrels? Of 100 barrels is 20 barrels? Of 1000 barrels is 200 barrels?

5. 300 is 50% more than what number?

6. 300 is 50% less than what number?

7. What number increased by 10% of itself is equal to 220?

8. Sold a watch for $150, and thereby gained 50%; what did it cost me?

9. Sold a watch for $150, and thereby lost 50%; what did it cost me?

10. 25% of 80 is 20% of what number?

11. 33⅓% of 90 is 3% of what number?

WRITTEN EXERCISES.

1. My expenses for 1876 were \$2500, which is 25% more than my expenses for 1875; what were my expenses for 1875?

2. Find the cost of a carriage sold for \$390, at a gain of 30%?

3. What % of 25 rods is $34\frac{2}{8}$ yards?

4. A yard-stick has worn $\frac{1}{4}$ of an inch too short; what % of a yard is it too short?

5. $1\frac{1}{2}$ per cent. of a merchant's capital is $12\frac{1}{2}$% more than \$480; what is his capital?

6. The rent of a store is \$320 per month, which is $\frac{1}{1\frac{1}{2}}$ of 10% of the cost of the building; what is the cost of the building?

7. Two merchants began business with \$3600 each; one gained $66\frac{2}{3}$%, and the other 75%; what amount was gained by each?

8. From a piece of cloth containing 45 yards there were sold two suits of 15 yards each; what per cent. of the cloth remained?

9. A farmer owns 120 sheep, which is 15% of the number owned by his neighbor; how many have they both?

10. There were 72 pupils in school on Monday, which was 80% of the attendance on Tuesday, and 90% of the attendance on Wednesday; what was the attendance on Tuesday and Wednesday?

11. In a school of 100 pupils, 28% remained at home on Monday, 10% on Tuesday, and 20% on Wednesday; what was the attendance for each day?

12. What sum of money increased by 40% of itself equals \$735?

APPLICATIONS OF PERCENTAGE.

The principles of percentage are applied to two general classes of problems:—

1st. Those in which *time* is not considered; as, *Profit* and *Loss*, *Commission* and *Brokerage*, etc.

2d. Those in which *time* enters into the calculation; as, *Interest*, *Discount*, *Banking*, etc.

A knowledge of the rules of percentage already given is sufficient to enable the pupil to solve all those problems in which *time* is not an element.

PROFIT AND LOSS.

Profit and **Loss** are terms applied to gains or losses in business transactions, and are generally reckoned at a certain per cent. on the cost.

ORAL EXERCISES.

1. A watch costing \$40 was sold at a gain of 10%; what was the gain, and what was the selling price?

ANALYSIS.—10% of \$40 = \$4, gain. \$40 + \$4 = \$44, or the selling price.

2. A cow costing \$75 was sold at a loss of $33\frac{1}{3}$%; for how much was she sold?

3. For what must I sell hay, bought at \$16 per ton, in order to gain $12\frac{1}{2}$%?

4. A carriage costing \$150 sold for \$120; what was the loss per cent.?

5. I buy cloth at \$4 per yard, and sell it at \$5 per yard; what % do I gain?

6. I sell goods for $12, that cost $6; what per cent. do I gain?

7. A carriage sold for $30 more than it cost, which was a gain of 10%; what was the cost?

8. I lose 5% by selling tea for 5 cents per lb. less than it cost me; what was the cost?

9. Sold a house at a gain of $500, clearing 25%; what did the house cost me?

10. Sold a house for $2500, and thereby gained 25%; what was the cost of the house?

11. Sold wheat at $1.60 per bushel, and thereby lost 20%; what was the cost?

12. A farmer bought a horse and carriage at auction, and sold them again for $315, thus gaining 5%; what did they cost him?

WRITTEN EXERCISES.

1. A farm costing $9650.75, was sold at a gain of $12\frac{1}{2}$%; what was the gain, and what did the farm sell for?

2. Had $1150.60 worth of goods damaged by water, so that I sold them at a loss of 15%; what did I get for them?

3. Bought a pair of horses at public sale for $275, and sold them immediately for $325; what % did I gain?

4. Sold goods for $90, which was $\frac{3}{4}$ of their cost; what % did I lose?

5. A dealer sold a quantity of flour at a profit of $315, and thus gained 8%; what did the flour cost him?

6. A merchant's outstanding accounts amount to $1148.37; if he loses 8% of that sum in collecting, how much does he receive?

7. Bought milk for 30 cents per gallon, and sold it at 12½ cents per qt.; what % did I gain?

8. Sold two houses for $4000 each; on one I gained 25%, and on the other I lost 25%; what did they both cost me?

9. Sold a horse for $240, and thereby lost 20%; for what should I have sold him in order to gain 20%?

10. At what price must a coat, costing $18, be marked in order to fall 10% and yet gain 10%?

ANALYSIS.—10% of $18 = $1.80. $18 + $1.80 = $19.80 = selling price at 10% gain. Now, as this is a reduction of 10% from the asking price, it follows that $19.80 = 90% of the asking price.

11. At what price must I mark a watch that cost $240, so that I can deduct 4% from the asking price and yet make 20% profit?

COMMISSION AND BROKERAGE.

Commission is a sum paid to a person who transacts business for another, being usually a certain percentage.

A **Commission Merchant, or Agent,** is one who buys or sells goods or transacts business for another.

A **Broker** is a person who buys and sells stocks, bills of credit, etc.; his percentage is called **Brokerage.**

The **Net Proceeds** of a sale is the amount remaining after the commission, expenses, etc. are deducted.

A **Consignment** is a quantity of merchandise sent to a commission merchant to be sold.

The **Consignor** is the person who sends the goods, and the **Consignee** is the person to whom the goods are sent.

ORAL EXERCISES.

1. An agent sold a set of maps for $20, on which he received 20% commission; what was the amount of his commission?

2. A collector received 25% commission for collecting a bill of $100; how much did he receive?

3. A dealer in real estate received $50 commission for selling a lot; what did he get for the lot, his commission being at the rate of 5%?

4. An auctioneer sold some household goods for $80, and returned $76 to the parties for whom he sold; what was the amount of his commission, and what rate % did he charge?

5. An agent selling books for $3 each, received $1 for every book he sold; what was the rate of commission?

6. A commission merchant sold a consignment of potatoes for $80; he charged 10% commission, and $2 for expenses; what were the net proceeds?

WRITTEN EXERCISES.

1. A commission merchant sold 1200 bushels of wheat, at $1.40 per bushel, 500 bushels of corn, at 90 cents per bushel, and 600 bushels of oats, at 50 cents per bushel; what was his commission, at the rate of $3\frac{1}{2}$%, and what were the net proceeds?

2. If I charge $2\frac{1}{2}$% for collecting $650, what amount must I pay over to my employer?

3. A commission merchant sold a consignment of grain for $1250.25; his commission was $2\frac{1}{2}$%, and charges for freight, cartage, etc., $84.55; what amount did the consignor receive?

4. A real estate agent sold a farm of 80 acres, at $120 per acre; what was his commission, at $\frac{3}{4}\%$?

5. An attorney received $82.01 for collecting $1640.20; what rate % did he charge?

6. A broker received $11 for buying a note of $4400; what was the rate %?

7. An auctioneer received $112.50 for selling a house, his commission being $2\frac{1}{2}\%$; for what did the house sell, and what were the net proceeds?

8. A tax collector, after deducting 5% commission, paid into the treasury $23750; what was the whole amount collected, and what was his percentage?

9. A merchant sent his agent $6060, which he invested in wool, at 50 cents per pound, first deducting 1% commission; how much wool did he buy?

ANALYSIS.—The agent cannot charge a commission on his own pay; the $6060 therefore, includes the amount invested in wool, and the agent's commission on that amount, consequently 100% or the amount invested + 1% commission = 101% which = $6060.

10. I send my agent in Cincinnati a consignment of boots and shoes, which he sells for $1850, at a commission of $2\frac{1}{2}\%$. He invests the net proceeds in sole leather, at 40 cents per lb., after deducting a commission of $\frac{1}{2}\%$ for buying; how much leather do I receive?

11. My agent in Baltimore sells my consignment of 640 barrels of flour, at $6.25 per bbl. He deducts his commission of 2%, and invests the net proceeds in corn, at 80 cents per bu., charging no commission thereon; how much corn do I receive?

12. A broker receives $3015 with instructions to invest in bank stock, after deducting his brokerage of $\frac{1}{2}\%$; what amount does he invest, and what is his brokerage?

STOCKS AND BONDS.

A **Corporation** is an association of persons authorized by law to transact business as a single individual.

Stock is the funds or capital of a corporation, and is named according to the character of the corporation; as, *Bank Stock, Oil Stock, Railroad Stock,* etc.

Shares are the equal parts into which a stock is divided, and persons who own shares of stock are called **Stockholders.**

The amount specified on a share of stock is called its **Face, Nominal,** or **Par Value.**

The **Market Value** of stock is what it will sell for.

Stocks are **at par** when they sell for their face value; **above par** or at a **Premium,** when they sell for more; and **below par** or at a **Discount,** when they sell for less than their face value.

In issuing shares of stock, payment in full is not always required at once, but a certain percentage is demanded from time to time. This percentage is called an **Installment.**

The **Gross Earnings** of a company or corporation are its entire receipts; the **Net Earnings** are what remains after the expenses are deducted.

A **Dividend** is a sum paid to stockholders out of the net earnings of a corporation.

An **Assessment** is a sum which stockholders are required to pay, to make up losses in the business of a corporation.

Note.—Dividends are usually made annually, semi-annually, or quarterly; and both dividends and assessments are generally a certain percentage of the face of the stock.

Bonds are written obligations, intended to secure the payment of borrowed money at a specified time.

Note.—When a corporation wishes to increase its capital by borrowing, or a government desires to raise money in the same manner, it issues bonds to secure payment to the lender. These bonds vary in amount, and are bought and sold like ordinary stocks, from which they differ in bearing interest.

Bonds receive different names according to their character; thus, U. S. or Government Bonds are issued by the U. S. Government, State Bonds by a State, and City, County, District, or Corporation Bonds, by a City, County, District, or Corporation, respectively.

U. S. Bonds are of several classes, bearing different rates of interest, and payable at different dates.

The following are the principal U. S. Bonds:—

6% BONDS.

6's of 1881, payable in 1881.

5-20's, redeemable in 5 years, and payable in 20 years from their date of issue.

5% BONDS.

10-40's, redeemable in 10 years, and payable in 40 years from their date of issue.

5's of 1881 (new), redeemable after 1881.

4½% BONDS.

4½'s of 1886 (new), redeemable after 1886.

4% BONDS.

4's of 1901 (new), redeemable after 1901.

The interest on the above bonds is payable in gold, semi-annually for the 5-20's and 10-40's, and quarterly for the 5's of 1881, 4½'s of 1886, and 4's of 1901.

A **Coupon** is a certificate of interest attached to a bond. It is detached and presented for payment when the interest is due.

State, city, corporation, and other bonds are made payable at a specified time, and bear various rates of interest. They are named according to the parties by whom they are issued, and their rate of interest; thus, City 6's, Georgia 7's, etc.

Note.—Reference is here made to *interest* in connection with bonds. This will be more fully treated hereafter, the following examples referring only to the buying and selling of bonds as stock.

ORAL EXERCISES.

1. If I buy stock at \$95 per share, whose par value is \$100, is it at a premium or a discount, and what %?

2. A stock whose par value is \$50 sells at \$60; is it at a premium or a discount, and what %?

3. When a stock is quoted at 8% premium, what is the market value of a share whose face is \$100?

4. When a stock is quoted at 10% discount, what is the market value of a share whose par is \$50?

5. What will a hundred-dollar government bond cost me, at a premium of 12%?

6. A bank declares a dividend of 3%; what will the stockholder receive who owns 5 shares, par value \$100?

7. A man buys stock at 80, and sells at 90; what % does he gain on his investment?

ANALYSIS.—If a man buys at 80, and sells at 90, he gains 10 on an investment of 80. 10 is $\frac{1}{8}$ of 80. $\frac{1}{8} = .12\frac{1}{2}$, or $12\frac{1}{2}\%$.

8. If I buy at 90, and sell at 120, what % do I gain?

9. When a bank stock, par \$100, is quoted at 25% premium, how many shares can I buy for \$500?

WRITTEN EXERCISES.

1. I have 80 shares of bank stock, par value $100, which pays a quarterly dividend of 2½%; what is my annual income from the stock?

Note.—When the par value of shares is not given, it is understood to be $100.

2. I buy three U. S. 5-20 bonds of $500 each, at a premium of 12½%; what do I pay for them?

3. I buy railroad stock, at $42 per share, and sell it at $28; what % do I lose on my investment, and at what % discount does the stock sell, par value being $50?

4. The net earnings of a street car company are $10000; it has issued 1200 shares of stock, par value $50; what % of dividend does it pay, and how much money does A receive, who owns 75 shares?

5. I wish to invest $6630 in city 6's, at 10½% premium; how many $1000 bonds can I buy for that money?

6. The market value of a certain oil stock is 28, nominal value 50; at what % discount is it?

7. What is the discount on $8000 Missouri 6's, selling at 92? What is the market value?

8. I paid $960 premium on 120 shares of canal stock; what was the % of premium?

9. A manufacturing company lost $30000 by fire; what rate of assessment must each stockholder pay, in order to make good the loss, the number of shares being 1500?

10. My broker buys me 40 shares of Pennsylvania R. R. stock, at 46, par value $50, brokerage ¼%; what does it cost me?

Note.—Brokerage is charged on the face value.

11. How many U. S. 5-20 bonds of $500 each, at 108, can I buy for $21600?

12. I buy 80 shares of oil stock, at $28, and sell it at $35; what % do I gain on my investment, and what is the entire amount gained?

13. A broker bought stock at 3% discount, and sold it at 12% premium, thereby gaining $900; how many shares did he purchase?

14. I sell six $500 government bonds, at 12½% premium, and invest the proceeds in railroad stock, at 25% discount; how many shares of the latter do I receive?

15. What will 5 shares of mining stock cost when quoted at 109?

16. My broker bought me 30 shares of railroad stock, at 84, and sold the same for me, at par; how much did I gain, the brokerage in each transaction being ¼%?

TAXES.

A **Tax** is a sum of money assessed upon persons or property, for public purposes.

A **Poll** or **Capitation Tax** is a tax upon the person of a citizen. It is a certain specified sum, varying in different localities, and assessed upon each person not exempt by law.

A **Property Tax** is a tax assessed upon property, and is a certain percentage of its valuation.

Property may be divided into two classes:—

Real Property or **Real Estate,** consisting of houses, lands, etc., and **Personal Property,** consisting of movable property, such as money, furniture, tools, etc.

WRITTEN EXERCISES.

1. A tax of $48650 is to be assessed upon a town; the valuation of the property being $2432500; what is the rate?

2. What is the amount of tax paid by a resident of the above town, his property, worth $24000, being assessed at $\frac{2}{3}$ of its value, and his poll tax being $1.50?

3. A tax rate of $2\frac{1}{2}$ cents on the dollar produces $65000; what is the assessed valuation of the property?

4. At a tax rate of 2 mills on the dollar, find the rate % and the amount of taxes paid on a property assessed at $2800.

5. The tax rate of a certain city is $2\frac{1}{4}$%, and the assessed valuation of property $200000000; what is the amount to be raised by taxation?

6. A school district wishes to raise such an amount by taxation as will secure $2850, for building purposes, after deducting the collector's commission of 5%; the assessed value of the property being $300000, what is the rate?

DUTIES OR CUSTOMS.

The revenue of the U. S. Government is derived from taxes on manufactures, licenses, etc., called *Internal Revenue*, and from *Duties* or *Customs*.

Duties or **Customs** are taxes upon imported goods.

These taxes are collected at an office established by the government, called a *Custom House*, and a place containing a custom house is called a *Port of Entry*.

Duties are of two kinds—*Specific* and *Ad Valorem*.

A **Specific Duty** is a certain tax imposed upon an article without regard to the cost.

An **Ad Valorem Duty** is a certain percentage of the value of the goods, according to the **Invoice.**

An **Invoice** is a bill of goods from the seller to the importer, with prices and charges annexed.

Before calculating duties, certain allowances are made if necessary; these are *Tare, Leakage,* and *Breakage.*

Tare is an allowance made for the weight of the box, cask, etc., containing the goods.

Leakage is an allowance for waste of liquors imported in casks, barrels, etc.

Breakage is an allowance for loss of liquors imported in bottles.

Note.—Actual breakage or leakage only is allowed; the allowance being determined by gauging or counting.

The entire weight or value of the goods before any deductions are made, is called the **Gross Weight,** or **Gross Value.**

The **Net Weight** or **Net Value,** is that which remains after the deductions are made.

WRITTEN EXERCISES.

1. A sugar refiner imports 80 hogsheads of sugar, the gross weight of each hhd. being 450 lb.; what amount of duty did he pay, at $2\frac{3}{4}$ cents per lb., the tare being $12\frac{1}{2}\%$?

2. A merchant imports 25 casks of French brandy, each cask invoiced at 36 gallons; what is the duty, at 50%, the price of the brandy being $4 per gallon, and an allowance of $1\frac{1}{2}\%$ being made for leakage?

3. I import 340 bags of Rio coffee, the gross weight of each being 55 lb.; what is the duty, at 5 cents per lb., the tare being 2%?

4. What is the duty, at $2\frac{1}{2}$ cents per lb., on 48 casks of raisins, gross weight 134 lb. each, tare 12%?

5. What is the duty on 250 tons of steel rails, at $1\frac{1}{4}$ cents per pound?

6. The duty, at 60%, on 4000 yds. of silk grenadine was $3600; what was the invoice price per yard?

7. The duty on 1200 yards of Brussels carpet, invoiced at $1.70 per yd. is $724; what is the rate %?

INSURANCE.

Insurance is indemnity against loss.

The principal kinds of Insurance are **Fire, Life, Marine, Accident,** and **Stock,** (cattle, horses, etc.)

A **Policy of Insurance** is a written contract between the insurers and the insured.

A **Premium** is an amount paid to secure an insurance, and is a certain percentage of the sum insured.

WRITTEN EXERCISES.

1. What premium, at 2%, must be paid for an insurance of $6400, on a house?

2. A merchant insures a ship and cargo, worth $96000, for $\frac{3}{4}$ of their value; what is the premium at $2\frac{1}{2}$%?

3. I take out a life insurance policy for $10000; what is my first premium at $3\frac{1}{2}$%?

4. My premium for an insurance of $4500 is $112.50; at what rate am I insured?

5. A property is insured for $2050, which covers $\frac{3}{4}$ of its value, and the premium thereon at $2\frac{1}{2}$%; what is the value of the property?

INTEREST.

Interest is the money which is paid for the use of money.

The **Principal** is the sum for the use of which interest is paid.

The **Rate** is the per cent. of the principal paid for any given time.

Note.—When no time is mentioned, *per annum*, or *by the year*, is understood.

The **Amount** is the sum of the principal and interest.

Simple Interest is the interest on the sum loaned, for the given time, at the given rate.

Legal Interest is the interest according to a certain rate per annum, fixed by law.

Note 1.—A higher rate of interest than that prescribed by law, is termed *usury*, and is prohibited by law.

Note 2.—When the rate per cent. is not named in notes, or other business documents, the *legal rate* must be taken.

Note 3.—In most of the States, and on debts due the United States, 6% is the legal rate, although in some States a higher rate may be agreed upon by special contract.

CASE I.

The principal, rate, and time, being given, to find the interest or amount.

ORAL EXERCISES.

1. What is the interest on \$100 for 5 years, at 5% per annum?

ANALYSIS.—5% of \$100 = \$5; which is the interest for 1 year; and the interest for 5 years is 5 times \$5 = \$25.

2. What is the interest of \$50 for 7 years, at 6%?

3. What is the interest of $60 for 5 months, at 1% a month?

4. What is the interest of $200 for 4 months, at 9% per annum?

ANALYSIS.—9% of $200 is $18, which is the interest for 1 year; and the interest for 4 months is $\frac{4}{12}$ or $\frac{1}{3}$ of $18 = $6.

5. At 2% a month, what will I pay for the use of $100 for 6 months? At 12% per annum?

6. What is the interest of $400 for 6 months, at 6% per annum? At $\frac{1}{2}$% a month?

7. At 5% a year for 10 years, what part of the principal is equal to the interest?

ANALYSIS.—At 5%, $\frac{5}{100}$ of the principal = the interest for 1 year; and for 10 years, 10 times $\frac{5}{100}$ or $\frac{1}{2}$ of the principal = the interest.

8. At 6% per annum for $16\frac{2}{3}$ years, what part of the principal equals the interest?

9. What is the interest of $500 for 1 year, at 10%? For 10 years?

10. I borrow $200 from a friend for 1 year, at 6%; what do I owe him at the end of the year?

11. What is the interest of $50 for 2 yr. 3 mo., at 8%? What is the amount?

12. I place $600 in a savings bank, which pays 4% per annum; what is due me at the end of one year?

13. Find the amount of $100 for 2 yr. 6 mo., at 4%. At 5%. At 6%. At 10%.

14. Find the interest of $60, at 6%, for 2 months. For 3 months. For 6 months. For 8 months. For 9 months.

15. I loan $500 for 1 yr. 4 mo., at 6%; what amount is *due me at the* expiration of that time?

16. What is the interest of 1 dollar for 20 years, at 5%?

WRITTEN EXERCISES.

1. What is the interest of $200.25 for 3 years, 5 months, at 8%?

$200.25
.08
16.0200
$3\frac{5}{12}$
$54.7350 Interest.

ANALYSIS.—8% or .08 of $200.25 = $16.02, which is the interest for 1 year. 3 yr. 5 mo. = $3\frac{5}{12}$ yr. If the interest for 1 year is $16.02, the interest for $3\frac{5}{12}$ yr. = $3\frac{5}{12}$ times $16.02 = $54.73\frac{1}{2}$.

2. What is the amount of $420 for 2 yr. 3 mo. 15 da., at 6%?

$420
.06
12)25.20
2.10
$27\frac{1}{2}$
57.75 Interest.
420.00
$477.75 Amount.

ANALYSIS.—The interest of $420 for 1 year at 6% = $25.20, which being divided by 12 gives $2.10 as the interest for 1 month. 2 yr. 3 mo. 15 da. = $27\frac{1}{2}$ months. If the interest for 1 month is $2.10, the interest for $27\frac{1}{2}$ months = $27\frac{1}{2}$ times $2.10 = $57.75, which being added to the principal, gives $477.75 as the amount.

Note.—It is customary among business men to consider 30 days as a month, and 12 months as a year. In transactions with the U. S. Government, however, 365 days are considered a year.

RULE.

Multiply the principal by the rate per cent. expressed decimally, and the product will be the interest for 1 year. If the time is in years and months, multiply this product by the time expressed in years and the fraction of a year. If the time is in years, months, and days, take $\frac{1}{12}$ of the year's interest, and multiply it by the time expressed in months and the fraction of a month.

3. What is the interest of $180.50 for 2 yr. 8 mo., at 8%?

4. What is the interest of $1200 for 3 yr. 4 mo. 15 da., at 7%?

5. What is the interest of $860.90 for 11 mo. 10 da., at 5%?

6. What is the amount of $800 for 5 yr. 6 mo. 20 da., at 6%?

7. What is the amount of $100 for 8 mo. 19 da., at 5%?

8. What is the interest of $50 for 1 yr. 10 mo., at $4\frac{1}{2}$%?

9. What is the interest of $1.00 for 2 yr. 3 mo. 5 da., at 6%? What is the amount?

10. What is the interest of $100 for 2 yr. 3 mo. 5 da., at 6%? What is the amount?

11. I borrow $1000, at 2% a month; how much do I owe at the end of 6 mo. 9 da.?

12. I loaned $580.75 from January 1, 1874, to January 15, 1876, at 6%; what amount was then due me?

13. What is the interest, at 7%, on $1200, from July 6, 1873, to October 10, 1876?

14. What is the amount of $750.60 for 1 yr. 1 mo. 1 da., at 5%?

15. What is the interest of $100, at 6%, for 16 yr. 8 mo.? At 5%, for 20 years?

16. What is the interest of $850, at 8%, for 12 yr. 6 mo.? At 10%, for 10 years?

17. I bought a house for $4000, and agreed to pay for it in 1 year and 9 months, with $4\frac{1}{2}$% interest; what amount had I to pay?

18. What is the difference between the interest of $500, at 6%, for 10 years, and at 10%, for 6 years?

BUSINESS METHOD.

ORAL EXERCISES.

1. What is the interest of $1 for 1 year, at 6%? For 1 day?

ANALYSIS.—The interest of $1 for 1 year, at 6%, is 6 cents, and for 1 day, or $\frac{1}{360}$ of a year, it is $\frac{1}{360}$ of 6 cents $= \frac{1}{60}$ of a cent.

2. If the interest of $1 for 1 day, at 6%, is $\frac{1}{60}$ of a cent, what is the interest of $1 for 30 days? For 60 days? For 90 days? For 180 days?

3. If the interest of $1 for 1 day, at 6%, is $\frac{1}{60}$ of a cent, what is the interest of $20 for 1 day, at 6%? Of $60? Of $120? Of $240? Of $300?

4. What is the interest of $30 for 120 days, at 6%?

ANALYSIS.—If the interest of $1 for 1 day is $\frac{1}{60}$ of a cent, the interest of $30 for 1 day is 30 times $\frac{1}{60}$ of a cent $= \frac{30}{60}$ or $\frac{1}{2}$ cent. If the interest of $30 for 1 day is $\frac{1}{2}$ cent, for 120 days it is 120 times $\frac{1}{2}$ cent $= \frac{120}{2}$ or 60 cents.

What is the interest, at 6%, of—

5. $120 for 90 days? For 20 days?

6. $300 for 60 days? For 180 days?

7. $180 for 15 days? For 45 days?

8. $240 for 50 days? For 30 days?

9. If the interest on a certain sum, at 6%, is $72, what would be the interest on the same sum, for the same time, at 5%?

ANALYSIS.—If the interest at 6% = $72, the interest at 5% = $\frac{5}{6}$ of $72 = $60.

10. What is the interest of $360 for 20 days, at 6%? At 3%? At 4%? At 7%? At 8%? At 10%?

11. What is the interest of $360 for 60 days, at 6%?

WRITTEN EXERCISES.

1. What is the interest of \$400 for 1 yr. 6 mo. 21 da., at 6%?

1 yr. 6 mo. 21 da. = 561 days.

$\frac{1}{60} \times 400 \times 561 = 3740$ cents = \$37.40.

ANALYSIS.—As the interest of \$1 for 1 day, at 6%, is $\frac{1}{60}$ of a cent, the interest of \$400 for 1 day is 400 times $\frac{1}{60}$ of a cent = $\frac{400}{60}$ cents, and for 561 days it is 561 times $\frac{400}{60} = 3740$ cents, or \$37.40.

Note.—It will be seen in the above operation that the number denoting the dollars has been multiplied by the number denoting the days, and the result divided by 60, giving the answer in cents; now, as $\frac{1}{60}$ of a cent is $\frac{1}{6000}$ of a dollar, dividing by 6000 would have given the answer in dollars; the general practice, however, is to divide by 6, pointing off *three* figures if there are only *dollars* in the dividend, and *five* figures if there are *dollars* and *cents*, thus giving the answer in dollars and cents. Thus, $\frac{\$400 \times 561}{6000} = \frac{224.400}{6} = \$37.40$.

RULE.

Multiply the principal by the number denoting the time in days, and divide the product by 6, pointing off three figures in the quotient when the principal is in dollars only, and five figures when there are cents in the principal.

Note.—For any other rate %, multiply the interest, at 6%, by the given rate, and divide by 6.

2. What is the interest of \$500 for 84 days, at 6%?

3. What is the amount of \$1600 for 60 days, at 6%?

4. What is the interest of \$184.66 for 1 yr. 6 mo. 10 da., at 6%? What is the amount?

5. What is the amount of \$1 for 360 days, at 6%?

6. What is the amount of \$785.90 for 30 days, at 6%?

7. At 6%, what is the interest of \$5000 for 5 yr. 5 mo. 5 da.? What is the amount?

8. At 6%, what is the interest of $850 for 8 mo.?

9. What is the amount of $50.50 for 1 yr. 3 mo., at 6%?

10. What is the amount of $300, at 6%, for 10 days?

11. I owe a merchant $540 and agree to pay him the debt in 30 days, with interest at 6%; how much shall I have to pay him?

12. I buy a property for which I agree to pay $1000 cash, $1000 in 3 months, and $1000 in 6 months, with 6% interest; what will the property have cost me when the final payment is made?

13. I borrow $500, January 13, 1875, and pay it July 5, 1875, with 6% interest; what does the use of the money cost me?

14. What is the amount of $420.80 from July 9, 1876, to December 25, 1876, at 6%?

15. What is the interest, at 6%, on $120, borrowed on Christmas, and paid on New Year's Day, counting both days?

16. If I borrow $320, May 1, 1877, and pay it with 6% interest December 31, 1877, how much do I pay?

17. What is the interest of $800 for 1 yr. 2 mo., at 8%?

1 yr. 2 mo. = 420 days.

$\frac{\$800 \times 420}{6} = \$56.00.$

$\frac{8}{6}$ of $56.00 = $74.66.

ANALYSIS.—We find the interest at 6% to be $56.00; and if the interest at 6% = $56.00, the interest at 8% = $\frac{8}{6}$ of $56.00 = $74.66.

18. What is the interest of $326 for 1 yr. 3 mo. 10 da., at 4½%? At 5%? At 8%? At 10%?

19. What is the interest of $96.50 for 90 days, at 3%? At 7%? At 12½%? At 15%?

20. A note dated March 30, 1870, was paid July 6, 1870, with interest at 7%; what was the amount?

CASE II.

The interest, or amount, and the rate and time, being given, to find the principal.

WRITTEN EXERCISES.

1. I wish to lend such a sum of money at 6%, for 3 years, as will yield me $54 interest.

Interest of $1 for 3 yr. = $.18.
$54 ÷ .18 = $300.

ANALYSIS.—We find that the interest of $1 for 3 years at 6% is $.18. Now, if $1 of principal produces $.18 interest, to produce $54 interest will take as many dollars of principal as .18 is contained times in 54 = 300, the number of dollars required.

RULE.

Divide the given interest by the interest of one dollar for the given time and rate; or, divide the given amount by the amount of one dollar for the given time and rate.

2. At 5%, what principal will produce $4.62½ in 1 yr. 6 mo. 15 da.?

3. At 6%, what principal will produce $300 interest in 16 yr. 8 mo.?

4. A man wishes to provide an annual income of $1000 for his wife; what sum must he put at interest, at 6%, to produce that sum?

5. What principal will amount to $490.50 in 1 yr. 6 mo., at 6%?

6. I desire to put a certain sum at interest, at 6%, for 18 months, so that at the end of that time I shall have $228; what sum must I invest?

7. *What* principal, at 8%, will amount to $180 in 1 year?

8. If I receive \$28.50 as the interest due on money lent at $1\frac{1}{2}\%$ a month for 4 months, what sum did I lend?

9. A church wishes to secure to the pastor an income of \$100 per month; what sum invested at 6% will yield that amount?

10. I have a mortgage of \$900 on my property, due in 18 months; how much must I invest now, at 5%, to amount in that time to the required sum?

CASE III.

The principal, interest, and time, being given, to find the rate.

WRITTEN EXERCISES.

1. At what rate will \$400 yield $\$30.66\frac{2}{3}$ in 1 yr. 3 mo. 10 da.?

The interest of \$400 for 1 yr. 3 mo. 10 da., at 1% = $\$5.11\frac{1}{9}$. $30.66\frac{2}{3} \div 5.11\frac{1}{9} = 6$.

ANALYSIS.—If \$400, at 1%, for 1 yr. 3 mo. 10 da., yields $\$5.11\frac{1}{9}$, to yield $\$30.66\frac{2}{3}$ for the same time will take as many % as $5.11\frac{1}{9}$ is contained times in $30.66\frac{2}{3}$, which = 6. The rate, therefore, is 6%.

RULE.

Divide the given interest by the interest of the given principal for the given time at one per cent.

2. At what rate will \$180 yield \$27 interest in 3 years?

3. At what rate will \$600 amount to \$684 in 2 yr. 4 mo.?

4. I lent \$500 for 2 yr. 4 mo., and received at the end of that time \$552.50 in full; what was the rate %?

5. *At what rate* will any sum double itself in 16 yr. *8 mo.?*

6. My semi-annual interest on ten 500-dollar government bonds is $150; what is the rate per cent.?

7. My annual income on $800 invested in Kansas, at the legal rate, is $80, while the same amount invested in New Jersey at the legal rate yields only $56; what is the difference in the rates?

8. At what rate will I have to invest $15000 so that it may yield me $300 quarterly?

CASE IV.

The principal, rate, and interest, being given, to find the time.

1. How long will it take $500 to gain $45, at 6%?

The interest of $500 for 1 year at 6% = $30.

$45 ÷ $30 = 1½. 1½ yr. = 1 yr. 6 mo.

ANALYSIS.—If $500 gains $30 in 1 year, at 6%, to gain $45 at the same rate will take as many years as $30 is contained times in $45, which = 1½. 1½ years = 1 yr. 6 mo.

RULE.

Divide the given interest by the interest of the given principal at the given rate for one year.

2. I lent a friend $1300, at 6%; at settlement he owed me $1339; how long did he have the money?

3. In what time will $1250 gain $500, at 7%?

4. In what time will $1920 amount to $2178, at 6%?

5. In what time will $500 double itself, at 6%? At 5%?

6. In what time will $1800 amount to $1964, at 4%?

7. A young man, on coming of age, received $1798.50 as the amount of a legacy of $1650, at 6%; how old was he when the money was invested?

COMPOUND INTEREST.

Compound Interest is interest on both principal and interest, if the latter is not paid when due.

Note.—Interest may be compounded at whatever time it is made payable, as, *annually, semi-annually,* etc. It is not considered usury, but it cannot be collected by law.

1. What is the interest of $500 for 3 years, compounded annually, at 6%?

Principal	$500
Interest for 1st year . . .	30
Amount, or second principal	530
Interest for 2d year . . .	31.80
Third principal	561.80
Interest for 3d year . . .	33.70
Final amount	595.50
Principal	500.00
Compound Interest . . .	$95.50

ANALYSIS.—In this example the interest being compounded *annually,* we find the interest for one year, and add it to the principal, forming a new principal, on which we calculate the next year's interest, and so on. The first principal subtracted from the final amount gives the compound interest.

RULE.

Find the amount of the given principal for the first period for which it is to be compounded; proceed with this amount as a new principal for the second period, and so on; the difference between the last amount and the given principal will be the compound interest.

Note.—When there is a *partial* period, the interest for that time is to be computed on the last amount and added to it.

2. What is the interest on $1400 for 2 years, compounded semi-annually, at 6%?

3. What is the amount of $400 for 1 yr. 6 mo., compounded quarterly, at 4%?

4. What is the interest of $1500 for 3 years, compounded annually, at 5%?

5. What is the interest of $850 for 3 yr. 2 mo. 15 da., compounded annually, at 6%?

PARTIAL PAYMENTS.

Partial Payments are payments in part, of a promissory note, bond, or similar obligation. When payments are made, the amount and date of each are written as receipts, on the back of the note.

A **Promissory Note** is a written promise to pay a certain sum of money at a specified time. It is sometimes called **a note of hand,** or simply a **note.**

ILLUSTRATION.—January 2, 1877, Henry Hand, of Reading, settles his account with William Morgan, of the same city. He finds that he owes the latter $500, which he is unable to pay immediately, and therefore gives Morgan his written promise to pay the amount due, with or without interest, and at such a time and place as may be agreed upon between them.

The promissory note in the above case might read as follows:—

Reading, Jan. 2, 1877.

$500.

Sixty days after date, I promise to pay to the order of William Morgan, five hundred dollars, with interest, without defalcation. Value received.

HENRY HAND.

The **Date** of a note is the time when it is drawn.

The **Time** of a note is the time for which it is drawn.

The **Maker** or **Drawer** of a note is the person who signs it.

The **Payee** is the person to whom the money is to be paid.

The **Face** of a note is the amount for which it is drawn.

The *Date* of the above note is Jan. 2, 1877, the *Time* is 60 days, the *Maker* or *Drawer* is Henry Hand, the *Payee* is William Morgan, and the *Face* of the note is $500.

An **Indorser** is a person who writes his name on the back of a note, and thereby makes himself responsible for its payment.

An **Indorsement** is anything written on the back of a note.

In order to make the payment of the above note more secure, Morgan requires that it shall be *Indorsed* by some responsible person. Jones consents to do this, and by writing his name on the back of the note becomes responsible for its payment.

When a note is made payable to *bearer*, or to the *order* of any person, it becomes **negotiable**; that is, it can be transferred or sold.

Note.—A bill or note, to be valid, is not confined to any set form of words. A promise to deliver or to be responsible for so much money is a good bill or note; but it must be exclusively and absolutely for the payment of money. In Pennsylvania, however, the law requires that the words "without defalcation" shall be inserted in a promissory note, and, for the protection of the payee, the note should contain the words "value received."

Promissory notes do not bear interest until after they are due, unless otherwise specified. If a note is paid before it is due, and afterwards comes into the hands of another holder, for value, the amount of the note can be collected from the maker at maturity. It is customary to write the amount of a note in figures, and also in words.

WRITTEN EXERCISES.

1. *Chicago, March 8, 1875.*

$600.

Six months after date I promise to pay to the order of E. Simpson, six hundred dollars, with interest. Value received.

JOHN MORTON.

This note was indorsed as follows: July 11, 1875, $50; November 21, 1875, $10; April 6, 1876, $120. What was the amount due at settlement, January 1, 1877?

Face of note	$600
Interest from March 8, 1875, to July 11, 1875 (4 mo. 3 da.)	12.30
Amount due July 11, 1875	612.30
First payment (exceeding the interest)	50.00
New principal	562.30
Interest on $562.30 from July 11, 1875, to November 21, 1875 (4 mo. 10 da.)	12.18
Second payment (less than interest) $10.	
Interest on $562.30 from November 21, 1875, to April 6, 1876 (4 mo. 15 da.)	12.65
Amount due April 6, 1876	587.13
Third payment $120 + second payment $10 (the two payments exceeding the interest due)	130.00
New principal	457.13
Interest from April 6, 1876, to January 1, 1877 (8 mo. 25 da.)	20.19
Amount due January 1, 1877	$477.32

ANALYSIS.—The interest on the face of the note from its date to the first payment we find to be $12.30, and, as the first payment exceeds the interest, we add this interest to the principal, and subtract the amount paid ($50), which gives us a new principal, $562.30. The interest on $562.30 from the first to the second payment is $12.18, which is greater than the second payment ($10); we therefore find the interest on $562.30 from the second to the third payment; this interest is $12.65, and, as the two payments now exceed the interest due, we subtract their sum ($130) from the amount of the last principal and the interest due, which gives us a new principal, $457.13. The interest on $457.13 from the third payment to January 1, 1877, is $20.19, which being added to $457.13 gives $477.32 as the amount due January 1, 1877.

The above method is the one which has been adopted by the Supreme Court of the United States, and is called the

UNITED STATES RULE.

Find the interest on the given principal from the date of the note to the date of the first payment, and if this payment equals or exceeds the interest, subtract it from the amount then due, and proceed in like manner with the remainder as a new principal.

If any payment is less than the interest due, find the interest on the last principal to the date of the next payment, and so continue until the sum of the payments equals or exceeds the interest due; then subtract the sum of the payments from the amount due, and proceed as before.

The following arrangement of the example just given will be found to be convenient in all cases under the United States Rule.

YR.	MO.	DA.	TIME.	PRINCIPAL	INTEREST.	AMOUNT.	PAYMENT.	BALANCE.
1875	3	8						
1875	7	11	4 mo. 3 da.	$600	$12.30	$612.30	$50	$562.30
1875	11	21	4 " 10 "	562.30	12.18	587.13	10	457.13
1876	4	6	4 " 15 "	562.30	12.65		120	
1877	1	1	8 " 25 "	457.13	20.19	477.32	None.	477.32

2. *Boston, May 20, 1875.*

$650.

One year after date I promise to pay to Charles Miller, or order, six hundred dollars, with interest, at 6%. Value received.

HENRY HOWARD.

Indorsements.—September 3, 1875, $35; December 18, 1875, $10; March 5, 1876, $200. What was due at settlement, May 20, 1876?

3. A note of $1600, dated January 4, 1874, was indorsed as follows:—May 12, 1874, $56.75; October 19, 1874, $25; February 19, 1875, $450. What was due June 1, 1875, interest at 8%?

4. A note of $750, dated April 4, 1875, had the following indorsements:—October 1, 1875, $15; June 4, 1876, $20; October 15, 1876, $300; February 20, 1877, $350. What was due August 1, 1877, at 6%?

5. A note of $400, dated May 12, 1875, and drawing interest at 6%, was indorsed as follows:—November 6, 1875, $10; July 9, 1876, $10; December 1, 1876, $175.50. What was due February 6, 1877?

In the settlement of notes and of interest-bearing accounts running a year or less, merchants often make use of the following

MERCANTILE RULE.

Find the amount of the note or debt from the time it begins to draw interest, to the time of settlement; also the amount of each payment, from its date to the time of settlement.

Add the amounts of the payments together, and subtract their sum from the amount of the note or debt.

6. A note of $850, dated Jan. 3, 1876, and drawing interest at 6%, has the following indorsements:—Mar. 7, 1876, $30; May 9, 1876, $60.50; Aug. 9, 1876, $25. What was due Dec. 19, 1876?

7. On a debt of $1100 bearing interest at 6% from Feb. 6, 1876, the following payments were made:—March 28, 1876, $100; June 19, 1876, $350; Aug. 30, 1876, $400; Nov. 14, 1876, $50. What was due Jan. 1, 1877?

PRESENT WORTH AND DISCOUNT.

The **Present Worth** of a debt payable at some future time without interest is that sum of money which, at legal interest for the given time, will amount to the debt.

Thus, the present worth of a debt of $106, due 1 year hence without interest, is $100; because $100 at 6% interest for 1 year will amount to $106.

True Discount is the difference between a debt bearing no interest and its present worth.

Commercial Discount is a deduction from the nominal price of an article, the face of a bill, &c., and is usually a certain percentage, without regard to time.

Bank Discount is interest paid in advance, and for three days more than the nominal time.

To find the present worth and true discount.

WRITTEN EXERCISES.

1. What is the present worth of $600, due 1 year hence, interest at 6%?

Amount of $1 for 1 yr. at 6% = $1.06.

$600 ÷ 1.06 = $566.037, Present Worth.

$600 − $566.037 = $33.963, Discount.

ANALYSIS.—The amount of $1 for 1 yr. at 6% is $1.06; the present worth, therefore, of $1.06 due 1 yr. hence is $1, and the present worth of $600 will be as many dollars as 1.06 is contained times in 600, or $566.037. $600 − $566.037 = $33.963 discount.

RULE.

Divide the debt by the amount of one dollar for the given time and rate; the quotient will be the present worth.

Subtract the present worth from the debt, and the remainder will be the discount.

2. What is the present worth of a legacy of $2000 payable in 18 months, interest at 6%?

3. Find the discount on $1344 due 2 years hence, interest at 6%.

4. What is the present worth of $750 due in 1 yr. 1 mo. 25 da., interest at 7%?

5. Find the discount on $1280 due in 7 mo., interest at 6%.

6. What is the difference between the interest and the discount on $525 due 10 mo. hence, at 6%?

7. I am offered $3000 cash for my house, or $3082.75 payable in 9 months without interest; how much shall I lose by accepting the latter proposition, money being worth 6%?

8. Find the present worth of $1800 payable in three equal installments without interest, in 6, 9, and 15 months respectively, money being worth 8%.

9. I buy goods for $1150 cash and sell them for $1224 on a credit of 4 months; do I gain or lose, and how much, interest being at 6%?

10. If I buy goods for $1200 on a credit of 4 months, and sell them for $1176.47 cash, do I gain or lose, and how much, interest being at 6%?

BANKING.

A **Bank** is an institution incorporated for the purpose of receiving deposits, lending money, or issuing bills, called bank-notes, for circulation as money.

Banks lend money on *promissory notes* made payable at a specified time. When a person has such a note discounted, he receives for it a sum equal to its face less the interest from the date of discount until its maturity.

When a bank discounts a note *drawing interest*, the discount is calculated on the amount of the note when due.

The **Proceeds** or **Avails** of a note is the amount received for it after being discounted, and is the face of the note less the bank discount.

Days of Grace are three additional days allowed for the payment of a note after the expiration of the time named therein.

A note is at **Maturity** and is legally due at the expiration of the days of grace.

The **Term of Discount** is the time from the date when the note is discounted until its maturity.

A **Protest** is a formal declaration made by a **Notary Public**, at the request of the payee or holder of a note, giving legal notice of its non-payment to the maker and the indorsers.

Note.—A protest for non-payment must be made upon the last of the three days of grace, unless that day should be Sunday or a legal holiday, in which case the protest must be on the day previous.

In discounting a note, banks include the day on which it is discounted and the day on which it matures. For example, a note drawn October 3, 1876, for 3 months, and discounted November 3, 1876, *matures* January 6, 1877, and the *term of discount* is from November 3 to January 6, both inclusive, or 65 days.

CASE I.

The face of a note, time, and rate, being given, to find the proceeds.

WRITTEN EXERCISES.

1. Find the proceeds of a note of $450, at 90 days, discounted at the Philadelphia Bank.

$450
93
1350
4050
6)41850
$6.975 Bank Discount.

$450
6.975
$443.025 Proceeds.

ANALYSIS.—As bank discount is simple interest at the legal rate for the time specified + 3 days of grace, we find the interest of $450 for 93 days = $6.98, which is the bank discount. Subtracting this from the face of the note gives us $443.02 as the proceeds.

RULE.

Find the interest on the face of the note for three days more than the time specified, and the result will be the bank discount.

Subtract the discount from the face of the note or sum discounted, and the remainder will be the proceeds.

Find the proceeds of each of the following notes:—

2. $300 for 60 days, discounted at 6%.

3. $1250 for 90 days, discounted at 8%.

4. $100 for 30 days, discounted at 6%.

5. $500 for 60 days, discounted at 6%.

6. $750 for 60 days, discounted at 8%.

7. Bought a lot of sugar for $320 cash, and sold it immediately, receiving in payment a note for $400 at 60 days, which I had discounted in bank at 6%; what did I gain in the transaction?

8. Find the maturity, term of discount, and proceeds of the following note:—

$450. *Philadelphia, Oct. 3, 1876.*

Three months after date I promise to pay to the order of Joseph Wood, three hundred dollars, at the People's Bank, without defalcation. Value received.

GEORGE MORTON.

Discounted Nov. 3.

Find the proceeds of each of the following:—

9. Note for $900, dated July 5, at 90 days; discounted Aug. 14, at 6%.

10. Note for $1325.25, dated July 22, at 60 days; discounted Aug. 1, at 7%.

11. Note for $4250.75, dated Sept. 29, at 3 months; discounted Oct. 3, at 9%.

12. Note for $1500, dated Sept. 1, at 4 months; discounted Sept. 25, at 6%.

13. Note for $365.15, dated May 3, at 90 days; discounted July 1, at 12%.

14. Note for $2000, dated Jan. 5, at 30 days; discounted Jan. 8, at 10%.

15. Note for $5000 *with interest* at 6%, dated June 4, at 90 days; discounted June 8, at 9%.

16. Note for $1776.95, dated Feb. 1, at 3 months; discounted Feb. 15, at 7%.

17. Note for $1900.05, dated Feb. 27, at 90 days; discounted Feb. 28, at 8%.

18. Note for $1000 *with interest* at 7%, dated March 8, at 60 days; discounted March 15, at 7%.

19. Note for $2500, dated June 1, at 90 days; discounted June 30, at 6%.

CASE II.

The proceeds of a note, time, and rate, being given, to find the face.

WRITTEN EXERCISES.

1. I wish to borrow $500 from bank; what must be the face of my note at 30 days, so that when discounted at 6% I shall receive that amount?

Discount on $1 for 33 days = $.0055.
$1 − $.0055 = $.9945 proceeds of $1.
$500 ÷ .9945 = $502.77 face of note.

ANALYSIS.—We find the proceeds of $1 for the given rate and time to be $.9945. Now, if it requires $1 to produce $.9945 proceeds, to produce $500 proceeds will require as many dollars as .9945 is contained times in 500, or $502.77.

RULE.

Divide the given proceeds by the proceeds of one dollar for the given time and rate; the quotient will be the face of the note.

2. What must be the face of a note at 90 days, that when discounted at 6% the proceeds may be $450?

3. If the proceeds are $1200, the time 60 days, and the rate of discount 6%, what must be the face of the note?

4. Bought a bill of goods amounting to $989.50; for how much must I give my note at 60 days, so that when discounted at 6% it will exactly pay the bill?

5. If I buy a house for $2910, how large a note at 6 months must I have discounted at 6%, in order to pay cash for it?

6. Find the face of a note at 30 days, which when discounted at 7% will yield $450.25.

INVESTMENTS.

CASE I.

To find the amount of income an investment will yield.

WRITTEN EXERCISES.

1. If I invest $21000 in a 6% stock, at 105, what will be the amount of my income?

ANALYSIS.—At 105, the cost of each $1 of stock is $1.05. $21000 ÷ 1.05 = $20000 = face of the stock. 6% of $20000 = $1200 income.

2. I invest $6480 in city 6's, at 108; what income does the investment yield me?

3. What income will $8000 yield me, if invested in Missouri 6's, at 95?

4. My broker invests $8950 in Maryland 5's, at 89, brokerage ½%; what income will the investment yield me?

ANALYSIS.—½% of $1 = ½ cent. $.89 + $.00½ = $.89½, the total cost of $1 of stock.

5. I invest $11000 in U. S. 10-40's, selling at 109¾, brokerage ¼%; what will be my income therefrom?

6. A invests $5600 in a 6% stock, at 112, and B the same amount in a 5% stock, at 80; which investment yields the greater income, and how much?

7. I sell a house which rents for $200 per annum, for $3300, and invest the proceeds in city 6's, at 110; is my income increased or diminished thereby, and how much?

8. What income will $5000 yield me if invested in New York 6's, at par?

9. What income will $10000 yield if invested in a 4% stock, at 65?

CASE II.

To find the rate per cent. of income an investment will yield.

WRITTEN EXERCISES.

1. If I buy 6 % bonds at 95, what is my rate of income?

ANALYSIS.—$1 of stock costs me $.95 and yields me $.06. $.06 \div .95 = .06\frac{6}{19}$, or $6\frac{6}{19}$ %.

2. What is the rate of income upon an investment in U. S. 10-40's, at 114?

3. I buy U. S. 5-20's, at 112; what rate of income do I realize on my investment?

4. What % will my money yield me if invested in city 6's, at $111\frac{3}{4}$, brokerage $\frac{1}{4}$ %?

ANALYSIS.—$\$1.11\frac{3}{4} + \$.00\frac{1}{4}$, brokerage = $1.12, the cost of $1 of stock.

5. I buy a house for $3000, which I rent at $20 per month; the taxes amount to $40 per annum, and the repairs to $20; what % do I get for my money, and how much greater % would the same amount have yielded me if invested in a 6 % stock, selling at 90?

6. If a stock paying 8 % dividend is selling at 8 % premium, what rate % of income will it yield as an investment?

7. If a stock paying 8 % dividend is selling at 8 % discount, what rate % of income will it yield as an investment?

8. What is my rate of income from a 10 % stock, purchased at 124?

9. Bought a stock at $13\frac{1}{2}$, which pays 2 % per annum; what is my rate of income?

10. What per cent. of income will a 5 % stock yield, if purchased at 110?

CASE III.

To find what amount must be invested, to yield a given income.

WRITTEN EXERCISES.

1. What sum must be invested in 6% railroad bonds, selling at 90, to obtain an income of $1200?

$1200 ÷ .06 = $20000, face of stock.
$.90 × 20000 = $18000.00, amount invested.

ANALYSIS.—As $1 of stock yields $.06 income, to yield $1200 income will require as many dollars of stock as .06 is contained times in $1200 = $20000; and as $1 of stock costs $.90, $20000 of stock will cost 20000 times $.90 = $18000, the amount invested.

2. If Philadelphia 6's are selling at 112½, what sum must be invested to secure an income of $800?

3. I wish to secure an annual income of $1800; what amount must I invest in U. S. 4½% bonds, selling at 108, to yield that income?

4. I have $2000 on interest at 6%; what sum must I invest in 8% school bonds, selling at 98, to yield the same annual income?

5. The U. S. 5-20 bonds of 1882 bear interest at 5%; if they sell at 109¾, how much must my broker invest for me to secure an annual income of $1000, brokerage ¼%?

ANALYSIS.—Each $1 of stock will cost me $1.09¾ + $.00¼ = $1.10.

6. What sum must I invest in a 7% stock, selling at 102, to pay the interest at 6% on an irredeemable ground rent of $1400?

7. What sum must be invested in a 10% stock, which is worth 140, to secure an annual income of $2500?

CASE IV.

To find the price at which a stock must be bought, to yield a given rate of income upon the investment.

WRITTEN EXERCISES.

1. At what price must I buy a 6 % stock, that it may yield 7 % income on the investment?

ANALYSIS.—The income of $1 of stock is $.06, which is 7 % of the price paid for $1 of the stock. $.06 \div .07 = .85\frac{5}{7}$: hence the stock must be bought at $85\frac{5}{7}$.

2. What must I pay for New York 6's, to realize 5 % income on the investment?

3. At what premium must I buy an 8 % stock, to obtain 6 % income on my investment?

4. I wish to secure 6 % income on a stock which pays 7 %; at what price must I purchase it?

5. At what rate would I have to purchase U. S. 10-40's, in order to secure 6 % income on the investment?

6. At what rate must I buy any 6 % stock, that it may yield me 6 % income on the amount invested?

7. What must I pay for a 6 % stock, that it may yield me 10 % annual income?

8. What must I pay for a 5 % stock, that it may yield $4\frac{1}{2}$ % income?

GOLD INVESTMENTS.

The term **Currency** is here used to denote the circulating medium employed instead of gold. When this circulating medium depreciates in value, gold is withdrawn from circulation, and becomes an object of investment, and is said to rise or fall as the value of the currency fluctuates.

CASE I.

To find the value of gold in currency.

WRITTEN EXERCISES.

1. How much currency is equal in value to $80 in gold, if the latter is quoted at 108?

ANALYSIS.—As $1 in gold is worth $1.08 in currency, $80 in gold is worth 80 times $1.08 = $86.40.

2. When gold was quoted at 250, what was the value in currency of $500 in gold?

3. If gold is 106, what is my income, in current funds, from a U. S. 10-40 bond of $1000?

4. I hold a ground rent of $500, the interest of which, at 6%, is payable in gold; how much currency will be required to pay the interest, if gold is 106¼?

5. I buy a bill of goods amounting to $16.20; how much currency should I get in change, if I tender a twenty-dollar gold piece in payment, gold being worth 108?

6. I hold a 6% mortgage for $1000, and a U. S. 5-20 bond for the same amount; how much greater income do I derive from the latter, when gold is at 115?

CASE II.

To find the value of currency in gold.

ORAL EXERCISES.

1. How much gold at 112½ can I buy for $50 in currency?

ANALYSIS.—As $1 of gold will buy $1.12½ of currency, for $50 of currency we can buy as many dollars of gold as $1.12½ is contained times in $50. $50 ÷ 1.12½ = $44.44, the required amount in gold.

2. What amount of gold, at 109⅓, can be bought for $250 currency?

3. I import a quantity of cigars, the duty on which is $175, payable in gold; how much currency will it require to purchase that amount, when gold is at 115?

4. How many watches, at $62½, in gold, can I buy for $1100 current funds, gold being at 110?

5. Bought goods for $1800, in gold, and sold them for $2500, in currency; what did I gain on the transaction, gold being at 120?

6. When gold was at 175, what was the value of $1 in currency?

REVIEW PROBLEMS.

1. A drover bought 120 sheep, at $4.50 each; 12½% of them were destroyed by a railroad collision, and the remainder were sold at $5.25 each; what did he gain by the transaction, the railroad company allowing him the cost of the sheep destroyed?

2. I purchase 30 boxes of Florida oranges, at $2 per box, each box containing 250 oranges; after losing 20% of them, I sell the remainder, at 25 cents per dozen; how much do I make by the transaction?

3. A merchant owes one man $1575.85, another $3460.72, and sundry other debts amounting to $5812.18; if his property is worth $7232.50, what per cent. of his indebtedness can he pay, and how much will the first man receive?

4. In an election the successful candidate received 20% more votes than his competitor; the whole number of votes cast was 8800; how many did each man receive?

5. A widow received $33\frac{1}{3}\%$ of an estate, the remainder being divided equally among her three children; what did each child receive, the widow's share amounting to $16000?

6. A hardware firm sold $250000 worth of goods in a year; $\frac{2}{5}$ of that amount was sold at a profit of 15%, and the remainder at a profit of 20%; what was the entire cost of the goods sold, and what was the profit?

7. A St. Louis grain dealer received $5112.50, with instructions to invest in corn, at 80 cents per bushel, after deducting his commission of $2\frac{1}{4}\%$; how much corn did he buy?

8. An agent sold 26 parlor organs at $180 each, and 12 melodeons at $75 each; he remitted $4185 to the manufacturers, retaining the balance as his commission; what was the rate of commission?

9. I buy a horse for $96, which I wish to sell at 30% profit; how much must I ask for him in order to throw off $5 and yet make the desired amount?

10. The gross earnings of a railroad company are $650000, and the expenses $350000; what rate % of dividend can be declared, if no surplus is reserved and the capital stock is $2400000?

11. My broker bought me 28 shares of bank stock at 102, brokerage $\frac{1}{4}\%$; I drew a dividend of 4% and then sold the stock at par; did I gain or lose on the transaction, and how much?

12. My house is insured at $\frac{3}{4}\%$ for $\frac{3}{4}$ of its value, the premium amounting to $21; what is the value of the house?

13. A merchant sold goods which cost $2500 at a gain of 20%, and another lot for $2500 at a loss of 20%. Did he gain or lose, and how much?

14. A tax collector pays into the treasurer's hands $47500, which is the amount of his duplicate less his commission of 5%; what is the tax rate if the assessed value of the property in his district is $2000000?

15. What amount of currency will be required to pay the duty on 28 hogsheads of molasses, 86 gallons each, leakage 1½%, duty 5 cents per gallon, the price of gold being 106½?

16. What is the duty, at 35%, on 50 dozen bottles of champagne, invoiced at 5 francs per bottle, breakage 5%, the franc being worth $.193?

17. *Charleston, May 20, 1875.*

$800.

Six months after date, I promise to pay to the order of James Rodgers, eight hundred dollars, with interest at 7%. Value received.

WILLIAM H. WESTER.

Indorsed as follows:—September 3, 1875, $30; December 23, 1875, $10; March 4, 1876, $180. What was due, by the United States rule, on settlement, August 15, 1876?

18. I received $1126.44 in payment of a loan at 6% interest, for 8 mo. 18 da.; what was the sum loaned?

19. Lent a friend $500, at 6%; when he repaid me it amounted to $545; how long had he the money?

20. If a bill of goods amounting to $1250 is bought on 4 months' credit, what amount of cash will settle the same at once, money being worth 6%?

21. A merchant bought goods for $1600 cash, and sold them at an advance of 12½%, receiving in pay a 30-day note, which was immediately discounted in bank, at 6%; what did he make on the transaction?

22. I bought a house for $3600; paid the real estate agent 1½% commission, the conveyancer $41.50, and a carpenter's bill of $21.80 for repairs; for how much must I draw a 60-day note, so that when discounted in bank at 6%, it shall cover the entire expense?

23. What sum must be invested in a 6% stock, selling at 95, to yield an income of $1680?

24. I bought a farm of 106 acres, at $150 per acre, and sold it a year later at an advance of $15 per acre; at what price would I have had to purchase a 6% stock, to yield me the same percentage on my money?

RATIO.

Ratio is the relation which one quantity bears to another of the same kind.

There are two kinds of ratio, *arithmetical* and *geometrical.*

Arithmetical Ratio expresses the difference between two quantities.

Geometrical Ratio expresses the division of one quantity by another.

Geometrical Ratio is indicated in two ways:—

By placing two dots between the numbers to be compared, considering the first number as a dividend, and the last as a divisor; or, by writing the two numbers in the form of a fraction. Thus, the ratio of 8 to 12 is written 8 : 12, and read 8 *is to* 12, or is expressed fractionally as $\frac{8}{12}$.

The quantities compared are called the **Terms** of the ratio; the first term being called the **Antecedent,** and the second term the **Consequent.**

A ratio may be either *direct* or *inverse*, *simple* or *compound*.

A **Direct Ratio** expresses the division of an antecedent by a consequent, and is the ratio usually meant when no direction is given to the contrary. Thus, if we speak of the ratio of 8 to 12, the direct ratio, or $\frac{8}{12}$, is the one intended.

An **Inverse Ratio** expresses the division of a consequent by an antecedent. Thus, the inverse ratio of 8 : 12 is $\frac{12}{8}$.

A **Simple Ratio** consists of one antecedent and one consequent; as, 2 : 4.

A **Compound Ratio** consists of the product of two or more simple ratios. Thus, the simple ratios of 8 : 12 and 2 : 4 give the compound ratio $8 \times 2 : 12 \times 4$.

In comparing fractional quantities, they must first be changed to a common denominator; when this is done, the fractions will be to each other in the ratio of their numerators. Thus, $\frac{5}{6} : \frac{8}{9} = \frac{30}{36} : \frac{32}{36} = 30 : 32$.

Since a ratio is expressed by a fraction, it follows that the terms of a ratio may be multiplied or divided in the same manner as the terms of a fraction, without changing the value of the ratio. Thus, the ratio 20 : 32 or $\frac{20}{32}$ may be changed to its lowest terms $\frac{5}{8}$, giving an equal ratio 5 : 8; or, it may be changed to any number of higher forms by multiplication, without altering the relative value of its terms.

ORAL EXERCISES.

1. What is the ratio of 8 to 4?

ANALYSIS.—$8 : 4 = \frac{8}{4} = 2$; or, the ratio of 8 : 4 is 2.

2. 2 is what part of 10; or, what is the ratio of 2 to 10? What is the ratio of 10 to 2?

3. $\frac{1}{4}$ is what part of $\frac{1}{2}$; or, what is the ratio of $\frac{1}{4}$ to $\frac{1}{2}$?

ANALYSIS.—$\frac{1}{2} = \frac{2}{4}$, and $\frac{1}{4} : \frac{2}{4}$ as $1 : 2 = \frac{1}{2}$.

4. $\frac{2}{3}$ is what part of $\frac{1}{6}$? $\frac{1}{6}$ is what part of $\frac{2}{3}$?

5. What is the ratio of $\frac{3}{4}$ to $\frac{7}{8}$?

6. If the antecedent is 4 and the ratio 2, what is the consequent?

ANALYSIS.—As the antecedent or dividend is 4, and the ratio or quotient is 2, the consequent or divisor must be $4 \div 2 = 2$, and the terms are 4 : 2.

7. If the consequent is 8 and the ratio 2, what is the antecedent?

ANALYSIS.—If the consequent or divisor is 8, and the ratio or quotient is 2, the antecedent or dividend must be $8 \times 2 = 16$, and the terms of the ratio are 16 : 8.

8. The numerator of a fraction is 8, and the value of the fraction is $\frac{1}{2}$; what is the denominator?

9. The denominator of a fraction is 16, and the value of the fraction is $\frac{1}{4}$; what is the numerator?

10. Find the value of each of the following ratios:—

18 : 3, 3 : 18, $\frac{18}{9} : \frac{9}{18}$, $\frac{5}{8} : \frac{3}{4}$.

WRITTEN EXERCISES.

1. Change the ratios 3 : 7 and 4 : 5 to a simple ratio.

$$\frac{3}{7} \times \frac{4}{5} = \frac{12}{35} = 12 : 35.$$

2. If the consequent is $\frac{3}{4}$ and the ratio $\frac{3}{4}$, what is the antecedent?

3. $\frac{1}{2}$ a peck is what part of a bushel, or what is the ratio of $\frac{1}{2}$ a peck to a bushel?

4. What is the inverse ratio of $4 : \frac{1}{2}$?

Note.—As a direct ratio is found by dividing the antecedent by the consequent, an inverse ratio is found by dividing the consequent by the antecedent.

5. 25 cents are what part of $1.50, or, what is the ratio of 25 cents to $1.50?

6. $\frac{3}{4}$ of a penny is what part of a shilling?

ANALYSIS.—In one shilling there are 12 pence, the ratio therefore is $\frac{3}{4} : 12 = \frac{3}{4} : \frac{48}{4}$ or $3 : 48 = \frac{3}{48}$ or $\frac{1}{16}$.

7. What is the ratio of 25 yards to 3 rods?

8. What is the ratio of 3 inches to 1 yard?

9. What part of £1 5 s. are 6 s., or what is the inverse ratio of £1 5 s. to 6 s.?

10. What part of 17 is $3\frac{1}{4}$? What is the ratio of $17 : 3\frac{1}{4}$?

11. What is the ratio of $\frac{1}{2} : 16\frac{1}{4}$?

12. What part of 1 square foot are $1\frac{1}{2}$ square yards?

13. What is the ratio of $1 to £1, the custom-house value of £1 being $4.8665?

14. What part of $1 is £1?

PROPORTION.

A **Proportion** expresses an equality of ratios.

Proportion is indicated by placing four dots between the two ratios, or by placing the sign of equality between them; thus, $4 : 8 :: 12 : 24$, or $4 : 8 = 12 : 24$. This proportion may be read, 4 is to 8 *as* 12 is to 24, or, the ratio of 4 to 8 is equal to the ratio of 12 to 24.

The first and last terms of a proportion are called the **Extremes,** and the second and third terms the **Means.**

In any proportion the product of the means is equal to the product of the extremes. Thus, in the proportion $4 : 8 :: 12 : 24$, $8 \times 12 = 4 \times 24$.

A **Simple Proportion** expresses the equality of two simple ratios, consisting of four terms, any three of which being given, the fourth can always be found.

ORAL EXERCISES.

1. If 2 pounds of butter cost 50 cents, what will 6 pounds cost?

ANALYSIS.—The ratio of the pounds is 6 : 2, and the consequent of the second ratio is 50 cents; the proportion will therefore be 6 : 2 :: what : 50. As the product of the means equals the product of the extremes, 6×50, or 300, is equal to 2 multiplied by the anteccdent 150, which is obtained by dividing 300 by 2; therefore 6 : 2 :: 150 cents : 50 cents. Cost of 6 pounds, $1.50.

2. 2 : 4 :: 6 : what?

ANALYSIS.—The product of the means is 24, and the quotient of $24 \div 2$ is 12; therefore 2 : 4 :: 6 : 12; or $\frac{2}{4} = \frac{6}{12}$.

3. 5 : 9 :: 10 : what? 4 : 8 :: 10 : what?

4. 2 : what :: 4 : 12?

ANALYSIS.—The product of the extremes is 24, and the quotient of $24 \div 4 = 6$; therefore 2 : 6 :: 4 : 12; or $\frac{2}{6} = \frac{4}{12}$.

5. If 3 pounds of sugar cost 30 cents, what will 10 pounds cost? 3 : 10 :: 30 : what?

6. What : 6 :: 8 : 24? $\frac{8}{24}$ = how many 6ths?

7. If 4 yards of cloth cost $16, what will 5 yards cost?

8. 8 : 10 :: what : 5? $\frac{8}{10}$ = how many 5ths?

9. The product of the means is 25, the first term is 5; what is the last term?

10. The product of the extremes is 24, the third term is 12; what is the second term?

11. If 3 barrels of apples cost $15, what will 12 barrels cost?

12. What : 6 :: 7 : 21? $\frac{7}{21}$ = how many 6ths?

13. $\frac{9}{27}$ = how many 9ths? 9 : 27 :: what : 9?

14. The numerator of a fraction is 12, and the value of the fraction is $\frac{1}{3}$; what is the denominator?

WRITTEN EXERCISES.

1. If 5 barrels of apples cost \$20, what will 100 barrels cost?

$$\frac{100}{5} \times 20 = \$400.$$

ANALYSIS.—As 100 barrels will cost more than 5 barrels, we express the ratio of 100 : 5, as an improper fraction $\frac{100}{5}$, and multiplying by the remaining or third term, 20, we obtain 400 as the number of dollars that 100 barrels of apples will cost.

2. If \$100 earn \$6 in a year, what will \$1500 earn in a year?

$$\frac{1500}{100} \times 6 = \$90.$$

ANALYSIS.—In this example it will be noticed that all three terms are of the same kind; but two of them are quantities which are said to *earn* something, whilst the third term is what is *earned* by one of the other two; we therefore make the two that earn, the terms of the ratio 1500 : 100, or $\frac{1500}{100}$, and multiply by the other term, 6, obtaining \$90 as the amount that \$1500 will earn in a year.

Note.—When the first and second terms are of different denominations, they must be changed to the same denomination; and the third term must in like manner be changed to the lowest denomination named in it. The answer will always be of the same denomination as the third term.

RULE.

Express the ratio of the terms which are of the same kind, as a proper fraction, when the answer requires to be less than the remaining term, and as an improper fraction, when the answer requires to be greater; multiply by the term which is of the same kind as the required answer, and the product will be the fourth term or answer.

3. If 10 gallons of wine cost \$42.50, what will 63 gallons cost?

4. The rent of my house is $200 for 5 months; what is the rent for a year?

5. The interest on my money for 3 months is $75; what is the interest on the same amount for 18 months?

6. Paid $67.50 for 9 tons of coal; what is the cost of 1 ton?

7. 3 bushels 2 pecks of potatoes cost $3.50; what is the cost of 1 peck?

8. 12 men can do a piece of work in 17 days; how long will it take 36 men to do the work?

9. $111.50 : $27.50 :: 33 : what?

10. 560 miles : 28 miles :: 20 hours : how many hours?

11. If $\frac{3}{4}$ of a dollar will purchase $\frac{1}{2}$ bushel of peaches, what will 5 bushels cost?

12. If $2\frac{3}{4}$ yd. of cloth cost $5.50, what will 1 yd. cost?

13. If £1 is worth $4.8665, what is the value of £7 6s.?

14. If a horse can travel 42 miles in 10 hours and 30 minutes, how far can he travel in 16 hours?

15. If six men can do a piece of work in 12 days when the days are 12 hours long, how long would they require to do the same work when the days are only 8 hours in length?

16. If $1\frac{1}{2}$ acres of land cost $600, what is the worth of $3\frac{3}{4}$ acres, at the same rate?

17. If a certain sum of money earns $25 in $3\frac{1}{2}$ months, how long will it require to earn $125?

18. 24 yards of carpet cost £16 10s.; what will 48 yards cost?

19. At the rate of $25 a month, what is the rent of a house for 4 years?

20. If $12\frac{1}{2}$ cents will buy $\frac{1}{4}$ lb. of tea, how much can be bought for $1.25?

COMPOUND PROPORTION.

A **Compound Proportion** expresses the equality of two ratios, one or both of which are compound.

ORAL EXERCISES.

1. If 4 men can dig a well in 8 days, how many such wells can 12 men dig in 16 days?

ANALYSIS.—As 12 men can dig more wells than 4 men, the ratio of the men is 12 : 4, or $\frac{12}{4}$. As more wells can be dug in 16 days than in 8 days, the ratio of the days is 16 : 8, or $\frac{16}{8}$. As the required answer is a number of wells, 1 well is the third term.

Multiplying the compound ratio $\frac{12}{4} \times \frac{16}{8}$ by 1, and cancelling common factors, we obtain 6 as the number of wells required.

2. If 2 men pay \$10 for 10 days' board, what should 4 men pay for 20 days' board?

3. If a 4-cent loaf weighs 8 oz., when flour is \$4 per barrel, what should an 8-cent loaf weigh, when flour is \$8 per barrel?

4. If \$10 earn 6 cents in 1 month, how much should \$20 earn in 6 months?

5. If I can buy 4 apples for 5 cents, when apples are \$4 a barrel, how many apples ought I to receive for 10 cents, when a barrel can be bought for \$2?

6. If a man can travel 100 miles in 10 days, walking 10 hours a day, how far can he travel in 5 days, walking 5 hours a day?

7. If 6 men can do a piece of work in 10 days, working 10 hours a day, how long will it require 10 men to do the same amount of work, if they work 3 hours a day?

8. $\left.\begin{matrix} 8 : 12 \\ 4 : 2 \end{matrix}\right\}$ :: 18 : what? $\frac{8}{12} \times \frac{4}{2}$ = how many 18ths?

WRITTEN EXERCISES.

1. If 1 man can saw 40 cords of wood in 10 days of 9 hours in length, how many cords can 4 men saw in 6 days of 10 hours in length?

$$\frac{6}{10} \times \frac{10}{9} \times \frac{4}{1} \times 40 = \frac{80}{3} = 106\frac{2}{3}, \text{ cords.}$$

ANALYSIS.—As less work can be done in 6 days than in 10, the ratio of the days is 6 : 10, or $\frac{6}{10}$. As in 10 hours more work can be done than in 9, the ratio of the hours is 10 : 9, or $\frac{10}{9}$. 4 men can do more work than 1, in the ratio of 4 : 1, or $\frac{4}{1}$. 40 cords is the remaining term, the answer being required in cords. Multiplying the product of the compound ratios by 40, and cancelling common factors, we obtain $106\frac{2}{3}$ as the number of cords.

RULE.

Express the ratio of each pair of terms that are of the same kind, as a proper fraction when the answer, considered with reference to that pair, requires to be less than the term which is of the same kind as the answer, and as an improper fraction when the answer requires to be greater, and multiply the product of these fractions by the term which is of the same kind as the answer.

2. 10 : 4, 6 : 2, 8 : 12 :: 24 : what?

14 : 9, 3 : 4, 15 : 7 :: 36 : what?

3. If 24 men in 9 days of 9 hours each, can dig a ditch 300 feet long, 3 feet wide, and 1 foot deep, how long will it take 60 men to dig a ditch 150 feet long, 2 feet wide, and 3 feet deep?

4. 40 : 65, 75 : 20, 32 : 8, 9 : 27, 14 : 7 :: 300 : what?

40 : 80, 30 : 50, 60 : 15, 25 : 100, 33 : 60 :: 125 : what?

5. If 2 barrels of flour will last 10 persons 7 weeks, how many persons would be required to consume 40 barrels of flour in 1 week?

6. If \$60 is the interest of \$1000 for 12 months, at 6 per cent., what is the interest of \$50 for 5 months, at 8 per cent.?

7. If \$25 gain 15 cents in 30 days, what will \$75 gain in 90 days?

8. If 12 men can earn \$240 in 20 days, working 9 hours a day, what should 36 men earn in 15 days, working 12 hours a day?

9. If 15 yards of silk will make 1 dress, the silk being 36 inches wide, how many yards will be required for 10 such dresses, if the width of the silk is 42 inches?

10. If 10 apples can be bought for 15 cents, when apples are worth \$4.50 a barrel, how many apples can be bought for 25 cents, when a barrel is worth \$5?

11. If cloth $\frac{3}{4}$ wide is worth \$3.75 a yard, what should 16 yards of similar quality cost, the width being $\frac{5}{4}$?

12. If the tax on a house worth \$2500 is \$56.25, at a rate of $2\frac{1}{4}\%$, what should the tax be on a house valued at \$15000, if the rate of taxation is $4\frac{1}{2}\%$?

13. If it costs \$6150 to carry 30750 tons 10 miles, how much should it cost to carry 500 tons 1000 miles, at the same rate?

14. If a family of 8 persons can use $3\frac{1}{2}$ gallons of milk in one week, in how many days, at the same rate, can a family of 16 persons use one half as much?

15. It requires 15 days to build a wall 60.5 ft. long, 18 in. thick, and 25.25 ft. high; how long will it require, at the same rate, to build a wall 120 ft. long, $12\frac{3}{4}$ in. thick, and $27\frac{1}{2}$ ft. high?

PARTNERSHIP.

A **Partnership** is an association of two or more persons for the purpose of transacting business.

A **Simple Partnership** is one in which each partner has his share of the capital employed for the same length of time.

A **Compound Partnership** is one in which the capital of the partners is employed for different lengths of time.

The **Capital,** or **Stock,** is the amount of property invested in the business.

The **Partners** are known as a **Firm,** or **Company,** and are frequently called a **House.** The profits or losses of the firm are divided among the partners according to each one's share in the business.

SIMPLE PARTNERSHIP.

ORAL EXERCISES.

1. A and B enter into partnership; A puts in \$1000, and B \$500; they gain \$100; what is the gain of each?

ANALYSIS.—The total capital was \$1500 and the gain \$100; A puts in \$1000, and is therefore entitled to $\frac{1000}{1500}$, or $\frac{2}{3}$ of \$100 $=\$66\frac{2}{3}$; B puts in \$500, and is therefore entitled to $\frac{500}{1500}$, or $\frac{1}{3}$ of $\$100=\$33\frac{1}{3}$.

2. Two boys buy 100 marbles; the first pays 4 cents, and the second 6 cents; how many marbles should each receive?

3. A and B own a house which rents for \$500 a year; A paid \$4000 as his share of the cost of the house, and B paid \$6000; how should the rent be divided between them?

4. A, B, and C pay \$1, \$2, and \$3, respectively, towards the purchase of some oranges; the oranges are sold for \$12; what should each receive for his share of the proceeds?

5. If 25 cents are divided between two boys so that one of them receives 3 cents as often as the other receives 2, how many cents will each boy receive?

6. A works 3 hours, B 4 hours, and C 5 hours, for which they receive \$7.20 in all; if they are paid at the same rate, what is each man's share?

WRITTEN EXERCISES.

1. A ship is valued at \$16000; A owns $\frac{3}{16}$, B $\frac{1}{4}$, C $\frac{1}{2}$, and the captain has the remaining share. The ship being injured in a storm, the repairs cost \$2560; what is each one's share of the expense?

ANALYSIS.—The value of the ship is \$16000, and the loss \$2560; \$2560 ÷ 16000, or $\$\frac{2560}{16000}$ = the loss on each dollar of capital invested; A owns $\frac{3}{16}$ of \$16000 = \$3000, B owns $\frac{1}{4}$ of \$16000 = \$4000, C owns $\frac{1}{2}$ of \$16000 = \$8000, and the captain owns $\frac{1}{16}$ of \$16000 = \$1000. A's loss will be $\$\frac{2560}{16000} \times 3000 = \$480$; B's loss, $\$\frac{2560}{16000} \times 4000 = \$640$; C's loss, $\$\frac{2560}{16000} \times 8000 = \$1280$; the captain's loss, $\$\frac{2560}{16000} \times 1000 = \$160$.

RULE.

Make the whole gain or loss the numerator, and the whole capital the denominator of a fraction; multiply this fraction by each partner's capital, for his share of the gain or loss.

2. A, B, and C agree to form a company for the purchase and sale of flour. A puts in \$1200, B, \$1800, and C, \$3000. At the expiration of three years they find that they are \$5000 in debt; what is each man's share of the indebtedness?

3. Two men purchase a piece of land for $1800. They sell it so as to gain $500; what is each man's share of the gain, if the first man paid twice as much as the second?

4. A property worth $20000 is to be divided in the proportion of $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$; what is the value of each share?

5. Four persons, A, B, C, and D, engage to perform a piece of work; A does twice as much as B, C does 3 times as much as B, and D does as much as B and C both. They receive $60; how much is each person's share?

6. A bankrupt, whose estate is worth $3000, owes $15000. What should A receive, his claim against the estate being $5000?

COMPOUND PARTNERSHIP.

ORAL EXERCISES.

1. Two men agree to build a wall for $20. The first man works 5 hours a day for 4 days, and the second man works 10 hours a day for 6 days; how should the money be divided between them?

ANALYSIS.—The first man works $5 \times 4 = 20$ hours; the second works $10 \times 6 = 60$ hours. The total number of hours' work is $60 + 20 = 80$. If 80 hours' work is worth $20, each man is entitled to as many 80ths of $20 as he has worked hours. The first man works 20 hours, and earns $\frac{20}{80}$, or $\frac{1}{4}$ of $\$20 = \$5$; and the second man works 60 hours, and earns $\frac{60}{80}$, or $\frac{3}{4}$ of $\$20 = \$15$.

2. John and James agree to do a piece of work for $5. John works 2 hours a day for 5 days, and James works 4 hours a day for 10 days; how much should each receive?

3. Two men agree to carry 100 pounds of groceries 5 miles, for $1. One of them carries 50 pounds 4 miles, the other carries his 50 pounds the entire distance, and the remaining 50 pounds 1 mile. How should they divide $1 between them?

WRITTEN EXERCISES.

1. A and B formed a partnership; A invests \$2000 for 6 months, and B \$1000 for 4 months; they gain \$1600; how much is each man's share of the gain?

A's capital = \$2000 × 6 = \$12000 for 1 month.

B's capital = \$1000 × 4 = \$4000 for 1 month.

\$12000 + \$4000 = \$16000, capital for 1 month.

$\$\frac{1600}{16000}$ = the amount gained by \$1 of capital.

$\$\frac{1600}{16000} \times 12000 = \$1200$, A's share of the gain.

$\$\frac{1600}{16000} \times 4000 = \$400$, B's share of the gain.

ANALYSIS.—If A's capital was employed but 1 month, it would require 6 times \$2000, or \$12000, to gain the amount earned by \$2000 in 6 months. If B's capital was employed but 1 month, it would require 4 times \$1000, or \$4000, to gain the amount earned by \$1000 in 4 months. It would therefore require \$16000 to gain as much in 1 month as \$2000 in 6 months and \$1000 in 4 months. If it requires \$16000 of capital to gain \$1600, \$1 of capital will gain $\frac{1}{16000}$ of $\$1600 = \$\frac{1600}{16000}$. A's \$12000 will gain $\$\frac{1600}{16000} \times 12000 = \$1200$; and B's \$4000 will gain $\$\frac{1600}{16000} \times 4000 = \$400$.

RULE.

Multiply each partner's capital by the time it is employed. Make the sum of all these products the denominator of a fraction, and the total gain or loss the numerator. Multiply this fraction by the product of each partner's capital by the time it is employed, and the result will be each partner's share of the gain or loss.

2. Messrs. Brown and Jones formed a partnership for 2 years. Brown paid in \$5000 at once, and Jones invested \$6000 at the beginning of the second year. At the end of 18 months, Brown withdrew \$1500. Their loss in the business was \$2500; how should it be divided between them?

3. A, B, and C formed a partnership. A invested $1000 for 6 mo., B $2000 for 9 mo., and C $900 for 12 mo. They gained $1000; how should it be divided among them?

4. The entire capital of a company is $6000. The gain in one year is $4800. John Thompson has invested $\frac{1}{3}$ of the capital during 6 mo., Amos Little $\frac{1}{2}$ the capital for 8 mo., and James Loomis the remainder for 12 mo.; what is each man's share of the gain?

5. Three men rented a farm for pasture, at a cost of $500. A has 50 head of cattle pastured for 4 mo., B has 200 head for 3 mo., and C has 300 head for $3\frac{1}{2}$ mo.; what should each man pay for his share of the rent?

6. January 1, 1876, John Jones commenced business with a capital of $2500. April 1, Lewis Smith became a partner with $3000 capital. July 1, Isaac Perkins entered the firm with $6000. January 1, 1877, the firm divided $3000; what did each partner receive?

7. Peter Carter enters a partnership six months after it is formed, and is required to invest sufficient capital to entitle him to $\frac{1}{4}$ of the profits. William Johnson has $2000 invested for 9 mo., and Samuel Davis has $6000 in the business during the entire year. They divide $4400; how much did Carter invest?

8. Three men agree to form a partnership for 18 months. A invests $\frac{1}{3}$ of the capital for 9 mo., and increases his proportion to $\frac{1}{2}$ the capital during the next 9 mo.; B invests $\frac{1}{4}$ of the capital for 6 mo., and then increases his share to $\frac{1}{3}$ for the remaining time; C invests $\frac{5}{12}$ of the capital during the first 6 mo., then withdraws $\frac{1}{5}$ of his share for 3 mo., and for the last 9 mo. he has but $\frac{1}{6}$ of the entire capital. How should they divide a profit of $2060?

EQUATION OF PAYMENTS.

Equation of Payments is the process of findin time when the payment of several sums of money d different periods may be made at once, without lo interest to debtor or creditor.

The **Equated Time** is the date at which the sum o debts may be paid without loss to either party.

The **Term of Credit** is the time between the con ing of a debt, and the date when it becomes due.

The **Average Term of Credit** is the time betwee contracting of the first debt, and the equated time fo payment of them all.

CASE I.

To find the equated time when the terms of c begin at the same date.

ORAL EXERCISES.

1. If the interest of \$1 for 1 day is $\frac{1}{60}$ of a cent, is the interest of \$60 for 1 day at the same rate?

2. At 6%, what will be the interest of \$1 for 60 d

3. What is the interest of \$10 for 12 days? Of \$ 120 days?

4. What is the interest of \$60 for 60 days? Of \$ 3600 days?

5. If the interest of \$1 for 1 month is $\frac{1}{2}$ cent, wh the interest of \$40 for 3 months? Of \$1 for 120 mo

6. What is the interest of \$100 for 5 months? C for 500 months?

7. If I borrow \$150 for 3 months, for how long sh I lend \$50 to balance the account?

8. If I lend $100 for 4 months, how long am I entitled to the use of $50 in repayment?

ANALYSIS.—The interest of $100 for 4 months is the same as the interest of $1 for 400 months; if I am entitled to the use of $1 for 400 months, I am entitled to the use of $50 for as many months as 50 is contained times in 400 = 8 months.

9. How long should I lend $100 to pay for the use of $200 for 2 months? For 4 months? For 8 months?

WRITTEN EXERCISES.

1. Samuel Jones bought goods of William Corson, to the amount of $1200, $400 of which is due in 3 months, and $800 in 4 months; what is the average term of credit?

$400 \times 3 = 1200$
$800 \times 4 = 3200$
1200)4400
$3\frac{2}{3}$ months.

ANALYSIS.—The interest of $400 for 3 months = the interest of $1 for 1200 months, and the interest of $800 for 4 months = the interest of $1 for 3200 months; hence the interest on the entire debt = the interest on $1 for 4400 months, or, the interest on $1200 for as many months as 1200 is contained times in 4400 months = $3\frac{2}{3}$ months, the average time of credit.

Note.—It will be seen in the above example that Jones has the use of $400 for $\frac{2}{3}$ of a month *after* it becomes due, while Corson, to balance this, has the use of $800 for $\frac{1}{3}$ of a month *before* it becomes due, that is, the use of *twice* as much money for *half* the time; hence the debt can be paid in $3\frac{2}{3}$ months, without loss to either party.

RULE.

Multiply each debt by the number denoting its term of credit, and divide the sum of the products by the number denoting the sum of the debts; the quotient will be the average term of credit. The average term of credit, added to the date of the debts, will give the equated time.

2. I buy $300 worth of goods on 3 months' credit, $500 worth on 4 months' credit, and $400 worth on 2 months' credit; what is the average term of credit?

3. What is the average term of credit of $1000 due in 6 months, $1400 in 8 months, and $1500 in 1 year?

4. Find the equated time of three bills of $600 each, bought January 3, 1877, on 2, 3, and 4 months respectively.

5. I owe William Lewis & Co. bills of $200, $600, and $400, for merchandise, bought on 1, 2, and 4 months respectively; when can I pay the entire amount without loss to either party, the goods having been bought May 3d?

6. What is the equated time for the payment of $450 due in 30 days, $400 due in 60 days, and $520 due in 90 days, the date of the bills being March 10th?

CASE II.

To find the equated time when partial payments have been made on a debt.

1. I owe a debt of $800 due in 8 months without interest; I pay $100 at the end of 2 months, and $200 at the end of 4 months; at what time should I pay the balance?

$100 \times 6 = 600$
$200 \times 4 = 800$
1400

$800 - 300 = 500$
$1400 \div 500 = 2\frac{4}{5}$.

ANALYSIS.—By paying $100 in 2 months, I lose the use of that sum for 6 months, and by paying $200 in 4 months, I lose the use of that sum for 4 months, or, by the two payments I lose the use of $1400 for 1 month; to balance this, I should keep the remainder of the amount due ($500) for as many months after maturity as 500 is contained times in 1400, or, $2\frac{4}{5}$ months.

RULE.

Multiply each payment by the number denoting the time it was paid before becoming due, and divide the sum of the products by the number denoting the balance unpaid; the quotient will be the time the balance should be kept after maturity.

2. I buy a house, January 3, for $4000, to be paid in 6 months without interest; if I pay $1000 March 3, and $1000 May 3, when, in equity, should I pay the balance?

3. I buy a bill of goods, September 5, at 90 days, amounting to $600; if I pay $300 in 30 days, at what date should I pay the balance?

4. A farmer buys a horse at a vendue for $120, on 4 months' credit; he pays, however, $40 cash; what should be the term of credit for the balance?

CASE III.

To find the equated time when the terms of credit begin at different dates.

1. What is the equated time of payment of the following bills: June 22, $500 at 60 days, July 3, $400 at 90 days, and August 5, $300, cash?

$500, June 22, at 60 days, due August 21.
$400, July 3, at 90 days, due October 1.
$300, August 5, cash, due August 5.

From Aug. 21 to Oct. 1 = 41 days.	500 × 41 =	20500
From Oct. 1 to Oct. 1 = 0 days.	400 × 0 =	00000
From Aug. 5 to Oct. 1 = 57 days.	300 × 57 =	17100
	1200	)37600
		31

October 1 − 31 days = August 31.

ANALYSIS.—If the entire debt had been paid October 1, the latest date on which any of the items becomes due, we find, by equating, that the debtor would have had the use of $1200 for 31 days after it was due. The date of payment, therefore, must be made 31 days before October 1, or August 31.

RULE.

Multiply each sum by the number denoting the difference in days between the date on which it becomes due, and the latest date on which any sum named in the account becomes due; divide the sum of the products by the sum of the debts, and the quotient will be the number of days to be counted backward from the latest date.

2. William Patterson bought goods of Evans & Co., as follows: May 10, $600, at 30 days, June 3, $400, at 30 days, August 9, $500, at 60 days; what is the equated time of payment?

3. A. W. Hahn bought merchandise of Robert Day, as follows:—

Aug. 3, a bill of $100, at 60 days.
" 9, " " $250, " 60 "
Sept. 3, " " $500, " 30 "
" 15, " " $200, for cash.

What is the equated time of payment?

4. Find the equated time of payment of the following bills: Jan. 3, $80, at 15 days, Jan. 10, $100, at 30 days, Jan. 19, $75, at 30 days, and Feb. 1, $60, at 10 days.

5. Bought the following bills of Hanly Bros., at 60 days: April 5, $150, April 15, $130, May 3, $225, May 20, $175; what is the equated time of payment?

6. Find the equated time of payment of the following bills: July 6, $150, at 90 days, Aug. 1, $200, at 60 days, Sept. 5, $180, at 30 days, and Sept. 25, $166, at 90 days.

AVERAGING OF ACCOUNTS.

To **Average an Account** is to determine the date when the difference between its two sides should be paid, or, to determine the amount of money which should be paid at a given date, without loss of interest to the debtor or the creditor in either case.

WRITTEN EXERCISES.

1. In the following account, when is the balance due, and what is the cash balance on the 30th of July, 1876?

Dr. HENRY WILLIAMS. *Cr.*

1876.				1876.			
Mar. 8	To Mdse. @ 4 mo.	$500	00	Apr. 3	By Cash	$380	00
May 3	" " 6 "	1001	00	June 1	"	800	00
June 7	" " 5 "	800	00	July 11	"	500	00

Dr.		*Cr.*	
Due July 8, $500×122=	61000	Due Apr. 3, $380×218=	82840
" Nov. 3, 1001× 4=	4004	" June 1, 800×159=	127200
" Nov. 7, 800× 0=	0000	" July 11, 500×119=	59500
$2301	65004 da.	$1680	269540 da.

$2301 − $1680 = $621, the balance due by the debtor.

269540 − 65004 = 204536 days' interest on $1 due to the debtor.

204536 ÷ 621 = 329, the number of days during which the debtor is entitled to the use of $621, after Nov. 7, or, till Oct. 2, 1877.

ANALYSIS.—We select November 7, the latest date on which any sum in the account becomes due, and reckon the number of days from this date to the date when each of the remaining items becomes cash, or due. We multiply each item by its number of days, and find the sum of the products on each side of the account. The difference of these sums, 204536, shows the number of days' interest on $1 due to the debtor. If it requires 204536 days for $1 to gain a certain interest, it will require $621, the balance of the account, $\frac{1}{621}$ of 204536 days, or 329 days, to gain an equal amount of interest. The date of payment should therefore be 329 days from November 7, or October 2, 1877.

RULE.

To find the equated time, multiply each item of debit or credit by the number which denotes the difference in days between the date on which it becomes due, and the latest date on which any item in the account becomes due.

Divide the difference between the sum of the debit and the sum of the credit products, by the difference between the sum of the debits and the sum of the credits, and the quotient will be the number of days to be counted forward from the latest date, when the larger sum of products is on the smaller side of the account, or backward when the larger sum of products is on the larger side of the account.

To find the cash balance on the 30th of July:—

Dr.

Due.	Days.	Items.	Interest.	Value.
1876.				
July 8	22	$500.00	+ $1.83	$501.83
Nov. 3	96	1001.00	— 16.02	984.98
Nov. 7	100	800.00	— 13.33	786.67
				$2273.48

Cr.

Due.	Days.	Items.	Interest.	Value.
1876.				
Apr. 3	118	$380.00	+ $7.47	$387.47
June 1	59	800.00	+ 7.87	807.87
July 11	19	500.00	+ 1.58	501.58
				$1696.92

$2273.48 — $1696.92 = $576.56, the cash balance required.

ANALYSIS.—We find the dates at which the several items are due on each side of the account, and compute the interest on each item for the number of days between its date and the date of settlement, July 30; adding the interest to the item when it is due before July 30, and subtracting it from the item when due after July 30. We then determine the *cash balance*, or difference between the two sides of the account, to be $576.56.

Note.—Interest has been computed by the business method; if exact interest is required, diminish each item of interest by $\frac{1}{7\frac{1}{3}}$ of itself.

RULE.

Find the number of days between the date when each item is due and the date of settlement, and the interest on each item for its number of days. Add the interest to the item when it is due before the date of settlement, and subtract the interest from the item when it is due after that date. The difference between the two sides of the account, thus corrected, is the cash balance required.

What is the equated time, and cash balance, in each of the following accounts, interest at 6%, the date of settlement in each being the 1st of November, 1876?

2.

Dr.	PETER SIMPSON.					*Cr.*			
1876					1876				
Aug. 1	To Mdse., at 90 da.	$750			Sept. 1	By Cash	$280	25	
" 30	" " 60 "	375	75		" 22	" Draft at 60 da.	800		
Sept. 8	" " 60 "	150	25		" 28	" Cash	50		
Oct. 30	" " 30 "	1500							

Note.—Allow three days of grace on notes and drafts.

3.

Dr.	OLIVER LADD.					*Cr.*			
1876					1876				
July 3	To Mdse., at 3 mo.	$1000			July 24	By Draft at 30 da.	$3000		
" 10	" " 60 da.	2500	75		Aug. 8	" Cash	500		
" 17	" " 30 da.	2000	75		" 15	" Note at 60 da.	1000		

ANALYSIS.

Every example in arithmetic can be performed by **Analysis**, that is, without the application of special rules; and it will be a profitable exercise to solve the preceding examples in Proportion, Partnership, and Equation of Payments, in this manner.

Note.—The teacher is not expected to exact the precise language of the solutions here given; an accurate statement from the pupil is all that should be required.

1. If 5 barrels of apples cost $25.25, what will 35 barrels cost?

ANALYSIS.—If 5 barrels of apples cost $25.25, 1 barrel will cost $\frac{1}{5}$ of $25.25, or $5.05; and 35 barrels will cost 35 times $5.05 = $176.75.

2. If $1\frac{3}{4}$ yards of cloth cost $3.50, what will $11\frac{3}{4}$ yards cost?

ANALYSIS.—If $1\frac{3}{4}$, or $\frac{7}{4}$ yards cost $3.50, $\frac{1}{4}$ will cost $\frac{1}{7}$ of $3.50 = $.50, and $11\frac{3}{4}$, or $\frac{47}{4}$ yards will cost 47 times $.50 = $23.50.

3. If 30 acres of land produce 1500 bushels of corn, how many bushels can be raised on $47\frac{1}{2}$ acres?

4. How many apples can be bought for $1.50, if 15 apples cost 25 cents?

5. If 6 table-spoons, containing 11 oz. of standard silver, can be bought for $23, what should 3 dozen spoons cost, if the amount of silver in each spoon is 2 oz.?

6. If 25 yd. of carpet cost $37.50, what should be the cost of 6 rolls of carpet, each containing 30 yards?

7. If the freight on a ton of coal is $1.50 for 100 miles, what should be the cost of 500 tons carried 456 miles?

8. An acre of wheat was sold for $15; what is the value, at the same rate, of a field of wheat containing 10 acres, 125 perches?

9. Bought a gallon of wine for $4.50; what is the value of 27 gal. 3 qt. 1 pt., at the same rate?

10. A ship sailed 600 miles in 5 days; how long would it require to sail 3000 miles, at the same rate?

11. If 24 lb. of iron cost 96 cents, what is a long ton of iron worth?

12. If $\frac{1}{3}$ of an acre of land produces $50 worth of fruit, what amount could be raised on $2\frac{1}{2}$ acres, at the same rate?

13. If my family can consume $1\frac{1}{3}$ barrels of flour in 3 weeks, how much flour would keep them for 1 common year?

14. If 6 men can do as much work as 12 boys, and 3 boys can do as much work as 2 women, how many women will it require to do as much work as 18 men?

15. 10 men can build a wall in 14 days; how long will it require for 3 men to build it?

16. A can do a piece of work in 5 days, and B can do the same work in 6 days; how long will it require for them both to perform the work?

ANALYSIS.—If A can do the work in 5 days, he can do $\frac{1}{5}$ of it in one day, and if B can do the work in 6 days, he can do $\frac{1}{6}$ of it in one day. Together they can do $\frac{1}{6} + \frac{1}{6} = \frac{11}{30}$ of it in one day; and if they can do $\frac{11}{30}$ in one day, to do $\frac{30}{30}$ or the whole work will take as many days as 11 is contained times in $30 = 2\frac{8}{11}$, the number of days required.

17. One pump can fill a cistern in 3 hours, another pump can fill it in $4\frac{1}{2}$ hours, and there is a delivery-pipe which can empty the cistern in 2 hours. If the cistern is empty and both pumps are set at work, with the delivery-pipe open, how long will it require to fill the cistern?

18. Three boys shared 120 marbles, in such manner that as often as the first received 3, the second received 4, and the third 5; how many did each receive?

19. $\frac{1}{4}$ of John's money exceeds $\frac{1}{5}$ of it by \$5; how much money has he?

20. A man starts in pursuit of a thief who is 3 miles away, and who travels at the rate of 4 miles an hour; the man travels $4\frac{1}{4}$ miles per hour; how long will he be in catching the thief?

21. A ship has three pumps; the first will free her from water in 1 hour after a leak is stopped; the second, in 2 hours; and the third, in $2\frac{1}{2}$ hours, the depth of the water being 6 feet. If the leak fills 6 feet of the vessel's hold in 3 hours, and the pumps are then set at work, how long will it require to free the ship from water, the leak still continuing?

22. How long will it require for two men to finish the remainder of a piece of work on which they have been engaged 5 days, if one of them could do the whole work in 20 days, and the other could finish the amount still remaining in 15 days?

23. If a pole 6 feet in height casts a shadow 5 feet in length, what is the height of a pole whose shadow is 40 feet 6 inches in length?

24. When flour is worth \$9 per barrel, what should be the weight of a five-cent loaf, if it weighs 6 ounces when flour is \$4.75 per barrel?

25. How shall \$90 be divided among 3 men, in the proportion of 1, 2, and 3?

ANALYSIS.—\$90 is to be divided into $1+2+3$, or 6 equal parts; $\frac{1}{6}$ of \$90 = \$15, $\frac{2}{6}$ of \$90 = \$30, and $\frac{3}{6}$ of \$90 = \$45. Hence the parts are \$15, \$30, and \$45.

26. Divide 150 into 3 parts, which shall be to one another as 3, 5, and 7?

27. In a mixture of 60 gallons, there are 4 parts of alcohol, 8 parts of water, and 3 parts of vinegar; how many gallons of each in the mixture?

28. An estate of $10000 was divided between two sons in the ratio of $\frac{1}{2}$ to $\frac{3}{4}$; what was the share of each?

29. Three schools containing 30, 40, and 50 pupils, respectively, are required to furnish $60 to purchase a library; what is the proportionate share of each school?

30. A quantity of gold was mixed in the proportion of 5 pwt. of 20 carats, and 6 pwt. of 18 carats; how much of each in 22 ounces of the mixture?

INVOLUTION.

Involution is the method of finding any power of a number.

A **Power** of a number is the number itself, or the product obtained by using the number two or more times as a factor.

A **Root** of a number is the number itself, or one of the equal factors by whose product the number is produced. Thus, 8 is the third power of 2, and 2 is the third or cube root of 8.

The **Exponent** of a power is a number placed at the right of the root and just above it, to show the number of times the root is to be used as a factor. It also denotes the degree of the power. Thus, 2^2 denotes the second power of 2, or 2×2; $2^3 = 2 \times 2 \times 2$, or the third power of 2.

The **Square** of a number, or its second power, is so called because the area of a square is obtained by the product of two equal factors, each of which represents the length of one of the sides of the square.

The **Cube** of a number, or its third power, is so called because the solidity of a cube is obtained by the product of three equal factors, each of which represents the length of one of the sides of the cube.

A **Parenthesis** ( ) is used to include such numbers as are to be considered together.

A **Vinculum** ‾‾‾ is sometimes used for the same purpose. Thus, $(10 + 5) \times (10 + 5)$, or $\overline{10+5} \times \overline{10+5}$ indicates that the sum of $10 + 5$ is to be multiplied by the sum of $10 + 5$, or $15 \times 15 = 225$.

ORAL EXERCISES.

1. What is the product of 2 used 3 times as a factor, or 2^3? What is the square of 6?

2. What is the third power of 3?

3. What is the cube of 4?

4. What is the value of 5^2?

5. Raise $\frac{1}{2}$ to the second power.

6. What is the cube of $\frac{1}{3}$?

7. The side of a square is 7 feet; how many square feet does it contain?

8. A box measures 5 feet in length, 5 in breadth, and 5 in depth: how many cubic feet does it contain?

9. What is the fourth power of 3?

10. Raise $\frac{1}{2}$ to the fourth power.

WRITTEN EXERCISES.

1. What is the third power of 25?

ANALYSIS.—We write 25 three times as a factor, and perform the multiplication. Thus, $25 \times 25 \times 25 = 625$, the third power of 25, or 25^3.

RULE.

Write the given number as many times as a factor as there are units in the exponent, or number expressing the required power; the product of these factors will be the required power.

2. Raise 36 to the fifth power.
3. What is the value of 6^5?
4. Find the cube of .175.
5. What is the square of $\frac{2}{15}$?
6. Find the cube of $\frac{1}{17}$.
7. Raise $2\frac{1}{4}$ to the fourth power.
8. What is the value of $3^2 \times 3^3$?

ANALYSIS.—$3^2 = 3 \times 3$, and $3^3 = 3 \times 3 \times 3$; $3^2 \times 3^3 = 3 \times 3 \times 3 \times 3 \times 3 = 3^5 = 243$.

Note.—As an exponent indicates a number of factors, it is evident that in the product of any number of powers of the same number there will be as many factors as there are units in the sum of the exponents; we can therefore represent the product of such powers by adding the exponents. Thus, $3^2 \times 3^3 = 3^{2+3} = 3^5$.

9. $6^2 \times 6^3 \times 6^4 \times 6^5$ = what power of 6?
10. What is the value of $4^4 \times 4^2 \times 4^3$?

INVOLUTION BY ANALYSIS.

1. Raise 15 to the second power by multiplying its tens and units separately.

ANALYSIS.—$15^2 = (10 + 5) \times (10 + 5)$.

$10 \times 10 = 100$ = the square of the tens.
$10 \times 5 = 50$
$10 \times 5 = 50$ } = twice the product of the tens by the units.
$5 \times 5 = 25$ = the square of the units.
225 = $(10 + 5) \times (10 + 5) = 15^2$.

This analysis indicates that *the square of a number consisting of tens and units is equal to the square of the tens, plus twice the product of the tens by the units, plus the square of the units.* To prove the accuracy of the proposition, let the tens of a number be denoted by a, and the units by b, then the square of the number will be denoted by $(a+b)\times(a+b)=a^2+2\,ab+b^2$; in which result a^2 = the square of the tens, $2ab$ = twice the product of the tens by the units, and b^2 = the square of the units. The inference drawn from the arithmetical analysis is therefore correct.

2. Raise 15 to the third power by multiplying its tens and units separately.

ANALYSIS.—$15^3=(10+5)\times(10+5)\times(10+5.)$

$10\times10\times10 = 1000$ = the cube of the tens.

$10\times10\times5$
$10\times10\times5$
$10\times10\times5$ } $= 1500$ = 3 times the square of the tens multiplied by the units.

$5\times5\times10$
$5\times5\times10$
$5\times5\times10$ } $= 750$ = 3 times the square of the units multiplied by the tens.

$5\times5\times5 = 125$ = the cube of the units.

$3375=(10+5)\times(10+5)\times(10+5)=15^3$.

In this operation we see that *the cube of a number consisting of tens and units is equal to the cube of the tens, plus three times the square of the tens multiplied by the units, plus three times the square of the units multiplied by the tens, plus the cube of the units.* To prove that this result is general, let a denote the tens and b the units of any number, then the cube of the number will be denoted by $(a+b)\times(a+b)\times(a+b)=a^3+3\,a^2b+3\,ab^2+b^3$; in which result a^3 = the cube of the tens, $3a^2b$ = three times the square of the tens multiplied by the units, $3ab^2$ = three times the square of the units multiplied by the tens, and b^3 = the cube of the units. The principle stated is therefore general in its application.

3. Raise the following numbers to the required powers by multiplying their tens and units separately:—

25^2, 55^2, 65^2, 99^2, 25^3, 45^3, 55^3.

EVOLUTION.

Evolution is the method of finding the root of any given power.

Roots are indicated by writing the **Radical Sign** $\surd$ before the number. Thus, $\surd 9$ denotes the square root of 9.

The **Index** of the root is a small figure placed over the radical sign to indicate the degree of the root. Thus, $\sqrt[3]{27}$ denotes the third or cube root of 27. In the square root the index 2 is usually omitted. Roots are sometimes indicated by a *fractional exponent*. Thus, $9^{\frac{1}{2}}$ denotes the square root of 9.

The **Square Root** of a number is one of the two equal factors by whose product the number is produced.

The **Cube Root** of a number is one of the three equal factors by whose product the number is produced.

Numbers that have exact roots are called **Perfect Powers**; all others are termed **Imperfect Powers.**

Roots that can be found exactly are called **Rational Roots**; all others are called **Irrational** or **Surd Roots.**

ORAL EXERCISES.

1. What is one of the two equal factors by whose product 9 is produced?

2. Name one of the two equal factors of 16.

3. Name one of the three equal factors of 8.

4. What is the square root of 25?

5. What is the square root of 36?

6. What is the cube root of 27?

7. What is the third root of 64?

8. The equal factors of 15625 are 25, 25, and 25; what root of 15625 is 25?

9. $406 \times 406 \times 406 = 66923416$; 406 is what root of 66923416?

10. What three equal factors will produce $\frac{1}{8}$ as their product?

11. What is the square root of $\frac{1}{4}$?

12. What is the cube root of $\frac{1}{27}$?

13. If 4 is taken 3 times as a factor, what is the value of the power, and what root of the power is 4?

14. What power of 10 is 10?

15. What root of 5 is 5?

16. What is the first root of 261?

17. What is the second root of 1?

18. What is the cube root of 1?

19. 64 is the product of two equal factors; what are the factors?

20. 64 is the product of 6 equal factors; what is one of the factors?

SQUARE ROOT.

If we take the least and the greatest numbers that can be expressed by one, two, three, or any number of figures, we find that their squares will contain either twice as many, or one less than twice as many figures as the roots. Thus, $1^2 = 1$, $9^2 = 81$, $10^2 = 100$, $99^2 = 9801$, etc.

From this we learn that there will be one figure in the root for every two figures in the second power, and an additional figure in the root for an odd figure in the power. Hence, in preparing a number for the extraction of the square root, we separate it into periods of two figures each, beginning at the right hand.

WRITTEN EXERCISES.

1. What is the square root of 225?

$$\begin{array}{r} \dot{2}\dot{2}5(10+5 \\ 10^2=100 \\ \hline 10\times 2=20)125 \\ 20+5=25\ 125 \\ \hline \end{array}$$

ANALYSIS.—We divide the given number into two periods, and find that the root will consist of two figures. As we found in Involution that $225 =$ the square of the tens, plus twice the product of the tens by the units, plus the square of the units in its root, we must find the square of the tens in the first left-hand period of the power. The greatest number of tens, whose square is contained in 2 hundreds, is 1. Subtracting the square of 1 ten, $(10^2) = 100$, from 225, there remains 125, which contains *twice* the product of the tens by the units of the root, plus the *square* of the units. Dividing 125 by 10×2 (twice the tens), we obtain 5 as the *probable* units' figure of the root. Adding the units' figure (5) to the trial divisor 20, we obtain the complete divisor $20 + 5$. As this divisor consists of twice the product of the tens plus the units of the root, multiplying by the units' figure (5) and completing the division we obtain 125, which equals twice the product of the tens by the units $(2 \times 10 \times 5)$ plus the square of the units (5^2); and the required root is $10 + 5 = 15$.

RULE.

Separate the given number into periods of two figures each, beginning at the right hand. Find the greatest number whose square is contained in the first left-hand period, for the first figure of the root. Subtract the square of this figure from the left-hand period, and to the remainder annex the next period for a dividend.

Double the root already found, for a trial divisor, and find how often it is contained in the dividend, excluding the right-hand figure. Write the quotient as the next figure of the root, annexing it also to the trial divisor as the units' figure of the complete divisor. Multiply the complete divisor

by the figure of the root just found, and subtract the pr from the dividend.

Double the root already found for a new trial di and continue the operation as before, till all the p have been used.

Note 1.—The left-hand period frequently contains but one fi

Note 2.—If any trial divisor is not contained in its dividend, a cipher to the root, a cipher to the trial divisor, and another to the dividend, and proceed as before.

Note 3.—If there is a remainder, after the root of a numb been extracted, annex periods of ciphers, and find the root required number of decimal places.

2. What is the square root of 2 to three decimal p

			2.000000(1.414
			1
Trial	Divisor	2	)100
Complete	"	24	96
Trial	"	28	)400
Complete	"	281	281
Trial	"	282	)11900
Complete	"	2824	11296
		Remainder	604

Note.—As we a quired to extract t to three decimal we annex three] of ciphers, pointi towards the *right*, a ceed as in whole nu For the benefit learner, we have the trial divisor, a as the complete d in each division.

PROOF.—Square the root 1.414 and add the remai the result should be 2.

3. What is the square root of 961?

4. Extract the square root of each of the foll numbers, 3025, 4356, and 6561.

5. What is the square root of 11881?

6. What is the value of $\sqrt{4016016}$?

7. Find the value of $\sqrt{810162.0081}$.

8. Extract the square root of $\frac{9}{16}$.

ANALYSIS.—$\sqrt{9}=3$, $\sqrt{16}=4$, $\sqrt{\frac{9}{16}}=\frac{3}{4}$.

Note.—The square root of a fraction can be found by extracting the square root of the numerator and of the denominator separately, or by changing it to a decimal and then extracting the root.

9. What is the value of each of the following: $\sqrt{\frac{4}{10}}$, $\sqrt{\frac{25}{49}}$, $\sqrt{\frac{36}{64}}$, $\sqrt{\frac{80}{144}}$, $\sqrt{\frac{121}{139}}$, $\sqrt{\frac{225}{625}}$?

10. Extract the square root of .1 to three decimal places.

11. What is the value of $\sqrt{.004}$ to three decimal places?

12. The area of a square lot is 1 acre; what is the length in yards of one of its equal sides?

13. What is one of the two equal factors of 15625?

14. I have a room in the form of a square, which requires 100 yards of carpet to cover it; what is the size of the room, if the carpet is 1 yard wide?

15. Find the side of a square whose area is 12 feet.

CUBE ROOT.

If we take the least and the greatest numbers that can be expressed by one, two, three, or any number of figures, we find that their cubes will contain either three times as many, or two or one less than three times as many figures as the roots. Thus, $1^3=1$, $9^3=729$, $10^3=10000$, $99^3=970299$, $100^3=1000000$, $999^3=997002999$, etc.

From this we learn that there will always be one figure in the root for every three figures in the power, and an additional figure in the root if there is a partial period of either one or two figures after the full periods have been pointed off. Hence, in preparing a number for the extraction of the cube root, we separate it into periods of three figures each, beginning at the right hand.

WRITTEN EXERCISES.

1. What is the cube root of 3375?

$$
\begin{array}{rrl}
 & & 3375(10+5 \\
 & 10^3 = & 1000 \\
\hline
10^2 \times 3 & = 300 &)2375 \\
10 \times 3 \times 5 & = 150 & \\
5^2 = & 25 & \\
\hline
 & 475 & 2375
\end{array}
$$

ANALYSIS.—We divide the given number into two periods, and find that the root will consist of two figures. As we found in Involution that 3375 = the cube of the tens, plus three times the square of the tens multiplied by the units, plus three times the square of the units multiplied by the tens, plus the cube of the units in its root, the cube of the tens must be found in the 3 thousands of the first period. The greatest number of tens whose cube is contained in 3 thousands is 1. Subtracting the cube of 1 ten, $(10^3) = 1000$, from 3375, there remains 2375, which contains three times the square of the tens multiplied by the units, three times the square of the units multiplied by the tens, and the cube of the units. Dividing 2375 by three times the square of the tens, or $10^2 \times 3 = 300$, as a trial divisor, the quotient is 7; but when we increase the trial divisor we find that 7 and 6 are both too great, and finally determine that 5 is the correct units' figure. We now add to the trial divisor three times the tens multiplied by the units $(10 \times 3 \times 5) = 150$, and the square of the units, $(5^2) = 25$, making the complete divisor $(10^2 \times 3) + (10 \times 3 \times 5) + (5^2) = 475$. Multiplying by the units' figure we obtain 2375, and find that $10 + 5$ or $15 =$ the cube root of 3375.

If we multiply the complete divisor in parts, as in the example given in Involution, we shall have $(10^2 \times 3 \times 5) + (10 \times 3 \times 5 \times 5) + (5^2 \times 5)$, an examination of which will show us that we have three times the square of the tens multiplied by the units, plus three times the tens multiplied by the square of the units, plus the cube of the units.

Note.—As any number consisting of two or more figures may be divided into a number of tens and units, the analysis given above will apply, no matter how many figures there may be in the root.

From the preceding analysis we derive the following

RULE.

Separate the given number into periods of three figures each, beginning at the right hand.

Find the greatest number whose cube is contained in the first left-hand period, for the first figure of the root. Subtract the cube of this figure from the left-hand period, and to the remainder annex the next period for a dividend.

For a trial divisor, annex one cipher to the figure of the root just found, square the number thus formed, and multiply it by 3, find how often it is contained in the new dividend, and the quotient will be the second figure of the root.

To the trial divisor add 3 times the product of the first figure of the root with a cipher annexed, by the quotient figure just found, and to the result add the square of the figure last found; the sum will be the complete divisor.

Multiply the complete divisor by the second figure of the root, and subtract the product from the dividend.

If there are any more periods to be brought down, annex the next period to the last remainder, and for a trial divisor annex a cipher to the figures of the root, square the number thus formed, multiply it by 3, and proceed as before.

Note 1.—If the trial divisor is not contained in the dividend, write a cipher in the root, annex two ciphers to the trial divisor, and bring down the next period for a new dividend.

Note 2.—If there should be a remainder after all the periods have been brought down, annex periods of ciphers, and find the root to any required number of decimal places.

Note 3.—If the given number contains a decimal, point off the periods to the right in the decimal, and, when any period is not complete, supply the deficiency with ciphers.

2. What is the cube root of 2 to three decimal pla

				2.000000000(1.2
	1^3	=	1	
Trial Divisor	$10^2 \times 3$	=	300)1000	
	$10 \times 3 \times 2$	=	60	
	2^2	=	4	
Complete Divisor			364	728
Trial Divisor	$120^2 \times 3$	=	43200)	272000
	$120 \times 3 \times 5$	=	1800	
	5^2	=	25	
Complete Divisor			45025	225125
Trial Divisor	$1250^2 \times 3$	=	4687500)	46875000
	$1250 \times 3 \times 9$	=	33750	
	9^2	=	81	
Complete Divisor			4721331	42491979
			Remainder	4383021

PROOF.—Cube the root 1.259 and add the remain the result should be 2.

3. What is the cube root of 997002999?

4. Extract the cube root of each of the follow numbers: 91125, 67917312, and 78402752.

5. Find the value of $\sqrt[3]{.8}$, $\sqrt[3]{.70}$, $\sqrt[3]{.125}$, and $\sqrt[3]{}$

6. Find the cube root of $\frac{4}{32}$.

ANALYSIS.—Changing the fraction to its lowest terms, $\frac{1}{8}$ find the $\sqrt[3]{1}=1$, and the $\sqrt[3]{8}=2$; therefore the $\sqrt[3]{\frac{1}{8}}=\frac{1}{2}$.

PROOF.—$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$.

7. Extract the cube root of each of the following

$\frac{2700}{6400}$, $5\frac{1}{8}$, .0065, 21.17, 18192.3145.

8. What is the cube root of 88998.7654?

9. Find the cube root of .0021 to 4 decimal places

10. What is the cube root of $\frac{125000}{216000}$?

11. If a cubical box holds a bushel of wheat, what is the depth of the box?

12. A can in the form of a cube holds just one gallon; what is the length of one of its sides?

13. What is the depth of a cubical vessel that will hold 10 barrels of water?

14. A water tank will contain 1500 gallons; what must be the length of the side in feet, if the form is that of a cube?

15. What is the length of a cubical pile of wood which contains 15 cords?

16. In excavating a cellar 24 ft. long, 15 ft. wide, and 6 ft. deep, 80 cubic yards of earth were removed; how deep would the cellar have been, if the three dimensions had been equal, the same quantity of earth being removed?

17. Find the value of each of the following:

$2 : 4 :: \sqrt[3]{64}$: what? $5 : 10 :: \sqrt[3]{125}$: the cube root of what?

Note.—The like sides of similar solids are to each other as the cube roots of their solidities. Thus, if the solidities of two cubes are 8 and 64, their sides are $\sqrt[3]{8}$ and $\sqrt[3]{64}$, or, if the ratio of the solidities is 8 : 64, the ratio of the sides is $\sqrt[3]{8} : \sqrt[3]{64}$.

18. If a cubical box contains 27 bushels, how many times larger must it be to contain 243 bushels?

19. If a coal-bin has a capacity of 64 tons, how many times larger must it be to contain 512 tons?

20. A box which measures 4 ft. long, 3 ft. wide, and $2\frac{1}{4}$ ft. in depth, contains 27 cubic feet; what are the sides of a similar box which contains 216 cubic feet?

21. There are two numbers which are to each other, as the $\sqrt[3]{\frac{1}{27}} : \sqrt[3]{343}$, and the less number is .05; what is the greater number?

MENSURATION.

Mensuration is the art of measuring lines, surfaces, and solids.

In measuring lines, surfaces, etc., surveyors make use of the following tables.

LINEAR MEASURE.

7.92 inches make 1 link, (l.)
25 links make 1 rod, (rd.)
4 rods make 1 chain, (ch.)
80 chains make 1 mile, (mi.)

Gunter's chain is 4 rods, or 66 feet, in length.

Measurements are recorded in *chains* and *hundredths*.

SQUARE MEASURE.

625 square links (sq. l.) make 1 pole, (P.)
16 poles make 1 square chain, (sq. ch.)
10 square chains make 1 acre, (A.)
640 acres make 1 square mile, (sq. mi.)
36 square miles make 1 township, (Tp.)

For the purpose of facilitating computations of the weight of materials used in the arts and trades, in farming, engineering, etc., the following tables have been prepared.

A Table showing the number of pounds in a bushel.

SUBSTANCE.	WEIGHT. POUNDS.	SUBSTANCE.	WEIGHT. POUNDS.
Barley	48	Peas	60
Coal, heaped bushel .	80	Potatoes	60
Clover seed . . .	60	Rye	56
Corn	56	Timothy seed . . .	45
Oats	32	Wheat	60

A Table showing the weight in pounds of a cubic foot of each substance named therein.

SUBSTANCE.	WEIGHT. POUNDS.	SUBSTANCE.	WEIGHT. POUNDS.
Alcohol, pure . .	49½	Lead	711
" common . .	52	Lignum-vitæ . .	83
Ale	65	Limestone . . .	197
Alum	107	Lime, quick . .	50
Anthracite, solid .	93	Linseed oil . .	58¾
" broken .	54	Manganese . . .	500
Bituminous coal, solid	80	Maple	47
" " broken	50	Marble	168
Brick, pressed . .	150	Mercury . . .	849
" common hard .	125	Milk	64½
" " soft	100	Nitre	119
Butter	59	Poplar	24
Cedar	35	Quartz	165
Cherry	44	Salt	133
Clay	125	Saltpetre . . .	131
Copper	547	Sand, dry . . .	94
Cork	15	" moist . .	112
Earth, solid	124	" wet . . .	130
" loose . . .	92	Silver	655
Glass, green . . .	165	Slate, average weight	175
Gold	1204	Steel, hard . . .	489
Granite	166	Tin	456
Gravel	120	Turpentine, spirits of	54
Ice	58	Vinegar	65½
Iron, cast	450	Walnut . . .	42
" wrought . . .	480	Water	62½
Lard	59	Wine, Burgundy	62

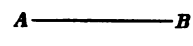
SUGGESTIONS TO TEACHERS.

Teachers who desire to make this subject attractive and instructive will do well to apply its principles in some practical manner. An empty barrel, and a gauger's rod, will furnish a better exercise for a class of scholars than whole pages of abstract examples.

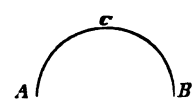
A visit to a public park in the city, or to the fields in the country, with a surveyor's chain, staff, and compass, requiring the pupils to make notes and measurements, for school work, will fix principles in the mind that would otherwise be soon forgotten. Measurements in the school-yard or play-ground will be of equal value. A neighboring pile of stone, or the excavation of a cellar, will furnish materials for better examples than any book can prescribe.

DEFINITIONS.

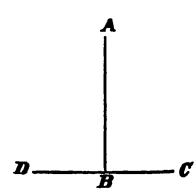
A **Straight Line** is a line that does not change its direction at any point; as the line *A B*.



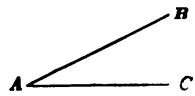
A **Curved Line** is one that is continually changing its direction; as the line *A C B*.



A **Perpendicular Line** is one that meets another, making the angles next to each other equal; thus, if the angles *A B C* and *A B D* are equal, the line *A B* is said to be perpendicular to the line *C D*.

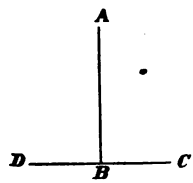


An **Angle** is a figure formed by two straight lines drawn from the same point; as the angle *B A C.*

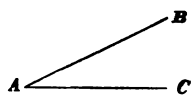


Note.—In speaking of an angle we name the letters or figures that stand at the ends of the lines which form it, placing in the middle the letter or figure which stands at the junction of the lines.

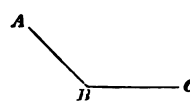
A **Right Angle** is either of the equal angles formed by two lines perpendicular to each other; as the angles *A B C* and *A B D.*



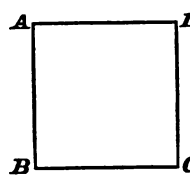
An **Acute Angle** is one that is less than a right angle; as the angle *B A C.*



An **Obtuse Angle** is one that is greater than a right angle; as the angle *A B C.*



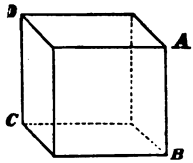
A **Surface** is the limit or boundary of a solid, and has only two dimensions, length and breadth; as the surface *A B C D.*



A **Plane Surface,** or a **Plane,** is a surface in which if any two points are taken, the straight line which joins these points will lie wholly in the surface.

A **Curved Surface** is a surface no part of which is plane.

The **Area** of a figure is the number of units of any required denomination included within its boundary line



A **Solid** is that which has length, breadth, and thickness; thus, the figure *A B C D* represents a solid.

The **Volume** of a solid is the number of units of any required denomination contained within the surfaces which bound it.

SURFACES.

A **Plane Figure** is a plane surface which derives other names from the nature of its boundary line or lines.

A **Polygon** is a plane figure bounded by straight lines.

A polygon of three sides is called a *Triangle;* of four sides, a *Quadrilateral;* of five sides, a *Pentagon;* of six sides, a *Hexagon,* etc.

TRIANGLES.

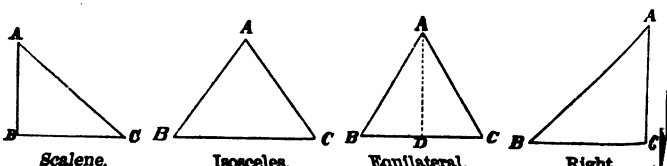
A **Triangle** is a plane figure bounded by three straight lines.

A **Scalene Triangle** has no two of its sides equal.

An **Isosceles Triangle** has two of its sides equal.

An **Equilateral Triangle** has its three sides equal.

A **Right Angled** or **Right Triangle** is one that has a right angle.



The **Base** of a triangle, or of any figure, is the side on which it is supposed to stand; and the **Altitude** is a line drawn from the angle opposite the base and perpendicular to it. Thus, *B C* is the base of either of the triangles immediately preceding, and *A D* is the altitude of the equilateral triangle *A B C*.

In a right triangle, the side opposite to the right angle is called the **Hypotenuse,** and the side which forms a right angle with the base is called the **Perpendicular.**

In the preceding right triangle, *A B* is the hypotenuse, and *A C*, the perpendicular.

To find the area of a triangle.

RULE.

From half the sum of the three sides, subtract each side separately; multiply the half sum and the three remainders together, and the square root of the product will be the area required.

Note.—When the base and altitude are given, the area is equal to the base multiplied by half the altitude.

1. What is the area of a triangular piece of ground whose sides are 3, 4, and 5 rods?

$\frac{3+4+5}{2}=6$

$6-3=3$

$6-4=2$ $\quad \sqrt{6\times3\times2\times1}=\sqrt{36}=6$ sq. rd., area.

$6-5=1$

2. What is the area of a triangle, the sides being 100, 150, and 200 rods?

3. The base of a triangle is 75 yards, and the altitude 20 feet; what is the area?

4. A triangular lot of ground is to be sold at the rate of $3 per square foot; what is the value of the lot, the sides being 150, 200, and 250 feet?

5. How many acres in a field whose sides are 350, 400, and 600 yards?

6. An irregular field whose sides are 20, 30, 40, and 50 rods respectively, measures 30 rods from opposite corners, dividing the first two sides from the third and fourth; what is the area of the field?

7. What is the area of a field, in the form of an isosceles triangle, the equal sides measuring 30 rods each, and the remaining side 20 rods?

8. The base of a right triangle is 27 chains, and the perpendicular is 36 chains; what is the area?

9. A field 160 feet in length, and 120 feet in width, is divided into two equal triangles by a line 200 feet in length, joining opposite corners; what is the area of the field?

Note.—It is established by Geometry that "The square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides." Hence the following:—

To find the hypotenuse of a right triangle.

RULE.

Extract the square root of the sum of the squares of the base and perpendicular.

To find the base or perpendicular.

RULE.

Extract the square root of the difference between the square of the hypotenuse and the square of the given side.

1. What is the distance from the top of a perpendicular tower 80 ft. in height, to a point 60 ft. from the base of the tower, the latter distance being measured on a line perpendicular to the side of the tower?

ANALYSIS.—The distance required is the hypotenuse of a right triangle, in which 60 ft. and 80 ft. are the lengths of the remaining sides. $\sqrt{60^2 + 80^2} = \sqrt{10000} = 100$ ft., hypotenuse.

2. Find the height of the tower in the last example, having given the hypotenuse 100 ft., and the base 60 ft.

ANALYSIS.—As the perpendicular equals the square root of the difference of the squares of the hypotenuse and base,

$$\sqrt{100^2 - 60^2} = \sqrt{6400} = 80 \text{ ft., height.}$$

3. A mason desiring to know whether the walls of his building were at right angles with each other, measured 12 yards on one wall, and 16 yards on the one which adjoined it, commencing both measurements at the same point and extending them horizontally; what should be the distance apart of the extreme ends of these lines, if the walls are at right angles?

4. Find the hypotenuse of a right triangle, the base being 48 yd. and the perpendicular 36 yd.

5. A flag-pole on the edge of a creek is 150 ft. high, and the distance from the top of the pole to the opposite edge is 250 ft.; what is the width of the creek?

6. What is the area of a right triangle, if the hypotenuse measures 120 ft. and the base 96 ft.?

7. What is the area of a right triangle, if the square of the hypotenuse is 288 chains, and the base and perpendicular are equal in length?

8. What is the area of a right triangle, if the base is 20 chains, and the hypotenuse 1 mile?

QUADRILATERALS.

A **Quadrilateral** is a plane figure bounded by four straight lines.

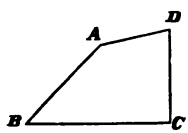
Quadrilaterals are divided into classes, as follows:—

A **Trapezium** is a quadrilateral which has no two of its sides parallel.

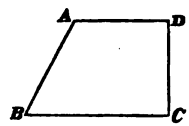
Note.—Two lines are *Parallel*, when they are situated in the same plane and have the same direction.

A **Trapezoid** is a quadrilateral which has only two sides parallel.

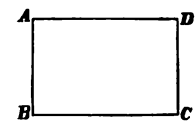
A **Parallelogram** is a quadrilateral which has two pair of parallel sides.



Trapezium.



Trapezoid.



Parallelogram.

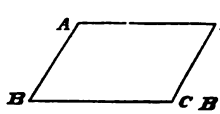
There are four kinds of parallelograms:—

A **Rhomboid** is a parallelogram whose adjacent sides are not equal, and whose angles are not right angles.

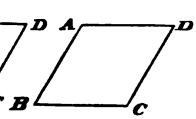
A **Rhombus** is a parallelogram whose sides are all equal, and whose angles are not right angles.

A **Rectangle** is a parallelogram whose angles are all right angles.

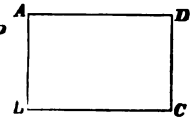
A **Square** is a parallelogram whose sides are all equal, and its angles all right angles.



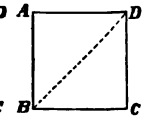
Rhomboid.



Rhombus.



Rectangle.



Square.

The **Altitude** of a parallelogram is the perpendicular distance between two of its parallel sides; as the line DE in the figure of the rhombus.

The **Diagonal** of a figure is a straight line joining its opposite corners; as the line DB in the figure of the square.

The **Perimeter** of a figure is the sum of all its sides.

To find the area of a parallelogram.

RULE.

Multiply the base by the altitude.

1. What is the area of a square field, the sides measuring 50 chains each?

$50 \times 50 = 2500$ square chains, area.

2. Find the area of a field in the form of a parallelogram, whose base is 40 rods, and altitude 25 rods.

3. How many acres in a piece of land in the form of a rhombus, the base being 30 chains, and the altitude 22 rods?

4. The base of a rhombus is 30 rods, and the altitude 150 yards; what is the area?

5. The base of a rhomboid is 375 ft. 6 in., and the perpendicular height 40 yards; what is its area?

To find the area of a trapezoid.

RULE.

Multiply half the sum of the parallel sides by the altitude.

1. What is the number of square feet in a trapezoid, one of the parallel sides being 30 inches, the other 4 feet, and the altitude 5 yards?

4 feet $= 48$ inches; $\frac{48+30}{2} = 39$ inches.

5 yards $= 180$ inches; $\frac{39 \times 180}{144} = 48\frac{3}{4}$ square feet, area.

2. How many square feet of surface in a board, the ends being parallel and measuring 18 inches and 9 inches respectively, the length being 12 feet?

3. What is the area of a trapezoid whose parallel sides are 225 and 330 yards, and altitude 125 feet?

4. What is the area of a trapezoid whose parallel sides are 20 yards and 16½ feet, and altitude 15 inches?

To find the area of a trapezium.

RULE.

Divide the trapezium into two triangles by a diagonal line; the sum of the areas of these triangles will be the area of the trapezium.

1. A meadow in the form of a trapezium, whose sides are 30, 40, 50, and 60 rods, measures 50 rods on a diagonal line dividing the first two sides from the others; what is the area of the field?

ANALYSIS.—Applying the rule for finding the area of a triangle when the three sides are given, we find the area of one of the triangles $= \sqrt{60 \times 30 \times 20 \times 10} = \sqrt{360000} = 600$ square rods; the other $= \sqrt{80 \times 30 \times 30 \times 20} = \sqrt{1440000} = 1200$ square rods; and the area of the trapezium $= 1200 + 600 = 1800$ square rods.

2. Given the diagonal of a quadrilateral field, 40 rods, and the altitudes of the triangles into which the diagonal divides the field, 120 feet and 75 feet respectively; what is the area of the field?

3. The sides of an irregular-shaped field are 15, 20, 25, and 30 rods, and the diagonal 100 yards; what is the area?

4. What is the area of a trapezium whose diagonal is 32 feet, the altitudes of the triangles into which the trapezium is divided being 12 and 16 feet respectively?

CIRCLES.

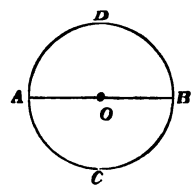
A **Circle** is a plane figure bounded by a curve, all the points of which are equally distant from a point within it, called the centre.

The **Circumference** is the curve which bounds the circle.

The **Radius** is any straight line drawn from the centre to the circumference.

The **Diameter** is any straight line drawn through the centre and terminated each way by the circumference.

In the figure, O is the centre, the curve $A\ B\ C\ D$ is the circumference, the circle is the space inside of the curved line, $O\ A$ and $O\ B$ are radii, $A\ B$ is a diameter.



To find the circumference of a circle, the diameter being given.

RULE.

Multiply the diameter by 3.1416.

1. The diameter of a wheel is 6 feet; what is the circumference?

$$6 \times 3.1416 = 18.8496 \text{ feet, circumference.}$$

2. If the diameter of the earth is 7960 miles, what is its circumference?

3. Find the circumference of a circular garden whose radius is 500 feet.

4. What is the circumference of a tree whose diameter is 30 feet?

To find the diameter when the circumference is given.

RULE.

Divide the circumference by 3.1416.

1. The circumference of a wheel is 18.8496 feet; what is its diameter?

$$18.8496 \div 3.1416 = 6 \text{ feet, diameter.}$$

2. The circumference of the earth is about 25000 miles; what is its diameter?

3. The circumference of a tree is 93 feet; what is its diameter?

4. What is the length of a string used to draw a circumference of 37.6992 inches?

To find the area of a circle, the diameter or the circumference being given.

RULES.

Multiply the square of the diameter by .7854; or, the square of the circumference by .07958.

1. The diameter of a pond is 50 feet; what is the area?

$$50 \times 50 \times .7854 = 1963.5 \text{ square feet, area.}$$

2. The circumference of a race-track is one mile; what is the area?

3. The diameter of a circular park is $\frac{3}{4}$ mile; what is the area?

4. The diameter of a circle is 400 feet, and the circumference 1256.64 feet; find the area from each dimension.

5. How many acres in a circular field whose radius is $\frac{1}{2}$ mile?

To find the diameter or the circumference of a circle, the area being given.

RULES.

Divide the area by .7854, and the square root of the quotient will be the diameter; or, divide the area by .07958, and the square root of the quotient will be the circumference.

1. The area of a circular pond is 1963.5 feet; what is the diameter?

$1963.5 \div .7854 = 2500$. $\sqrt{2500} = 50$ feet, diameter.

2. The area of a circular field is 1 acre; what is the circumference?

3. What is the radius of a circle whose area is one square mile?

4. The area of a circle is 3 acres, 42 rods; what is the diameter?

5. The length of a fence is required that will exactly surround a circle containing one-half a square mile.

6. The area of a circle is 7854 square yards; required the diameter and the circumference.

To find the side of an inscribed square, the diameter or the circumference of the circle being known.

Note.—A square is *inscribed* in a circle when its diagonal is one of the diameters of the circle. The circle is then said to be *circumscribed* about the square.

RULES.

Multiply the given diameter by .7071; or, the given circumference by .22507.

1. The diameter of a log of walnut is three feet; how large a square stick can be hewn from it?

$3 \times .7071 = 2.1213$ feet square.

2. The circumference of a tree, exclusive of the bark, is 22.507 ft.; what is the size of the largest piece of square timber that can be cut from it?

3. What was the diameter of a log which measures 7.071 ft. across the corners after it has been squared?

4. The side of a square field is 50 yd.; what is the circumference of the circumscribed circle?

5. The area of a circular field is one acre; what is the side of the inscribed square?

REVIEW PROBLEMS.

1. What is the length in rods of the side of a square whose area is 1 acre?

ANALYSIS.—As the area is the product of two equal factors, we change 1 acre to rods, and extract the square root of the number thus obtained. 1 A. = 160 rods. $\sqrt{160} = 12.649+$ rods.

Note.—As the area of any surface is the product of two factors, it is only necessary to divide the area by either factor in order to find the other.

2. What is the length of a field containing 9 acres, the form being that of a rectangle, and the width 16 rods?

3. The base of a triangular field is 20 rods, and the area 5 acres; what is the altitude of the triangle?

4. A field in the form of a rectangle is divided into two triangles by a diagonal line which measures 10 rods; the sides of the rectangle are 6 and 8 rods, respectively; what is the area of one of the triangles?

5. The circumference of a wheel is 3.1416 yards; what is the distance from the centre to any point in the circumference?

6. One side of a field in the form of a parallelogram is 20 rods, and the area is 10 acres; what is the other side?

7. The length of a line by which a circle is inclosed is 75 yards; what is the length of a line inclosing a circle which contains 4 times the area of the first?

8. A farmer has two fields, each containing 1 acre. The first is in the form of a rectangle, one side of which is 40 yards; the second is a square field. What is the length of the fence which surrounds each field?

9. What is the value of a farm in the form of a trapezoid, the parallel sides of which measure 80 and 31 chains respectively, the perpendicular distance between these sides being 120 rods; the land being worth $15 an acre?

10. A circular piece of ground has a square laid off within it, the area of which is $\frac{1}{2}$ acre; what is the diameter of the circle, if the corners of the square touch the circumference?

11. How much farther will a horse have to run in going round the sides of a square mile of land, than in going round the same area in the form of a circle?

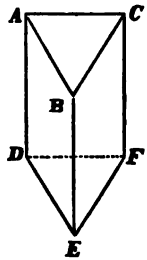
12. Two pieces of land measure each 1 mile around. What is the difference in area, if one piece is a square, and the other a rectangle, one side of which is 100 rods in length?

13. A ladder 60 feet long is placed against a house which is 50 feet in height; the foot of the ladder rests on the ground 36 feet from the house; how far is the top of the ladder from the top of the house?

14. What is the difference in area between a circle whose diameter is one rod, and a square whose diagonal is one rod?

SOLIDS.

A **Prism** is a solid, two of whose faces are equal polygons lying parallel to each other, and the remaining faces parallelograms. The parallel polygons are called the **Bases** of the prism; the parallelograms taken together constitute the **Convex Surface.**

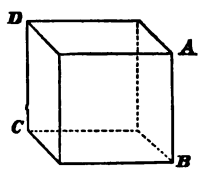


Triangular Prism.

Prisms are named from the form of their bases; thus, a prism whose base is a triangle is called **a Triangular Prism.**

A **Right Prism** is one whose edges are perpendicular to the planes of its bases.

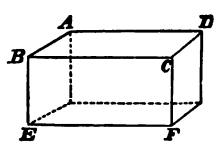
A **Cube** is a prism whose faces are all equal squares.



Cube.

A **Parallelopipedon** is a prism whose bases are parallelograms.

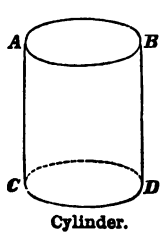
A **Cylinder** is a round body of uniform diameter, with equal circular bases, parallel to each other.



Parallelopipedon.

The **Altitude** of a prism or cylinder is the perpendicular distance between its bases.

The **Convex Surface** of a cylinder is the whole curved surface.



Cylinder.

The **Whole Surface** includes the convex surface and the surface of the two bases.

Similar Solids are those which have the same number of similar faces similarly situated.

To find the surface of a prism or cylinder.

RULE.

Multiply the perimeter of the base by the altitude for the convex surface, and to the product add the areas of the two ends.

1. What is the wall surface in a square room whose sides are 10 feet wide and 10 feet in height?

$(10 \times 4) \times 10 = 400$ square feet of wall surface.

2. The diameter of a circular column is 4 feet; what number of square yards of paper will be required to cover it, the height of the column being 15 feet?

$4 \times 3.1416 \times 15 = 188.496$ square feet; $188.496 \div 9 = 20.944+$ square yards of paper.

3. What is the convex surface of an octagonal pillar, each side of the base being 1.8 feet, and the height 12 feet?

4. The side of a cubical box is 3 feet; what is the whole surface?

5. How many square feet of boards will it require to cover the entire outside of a boiler, whose diameter is 3.5 feet, and length 20 feet, the boiler being a cylinder?

6. What is the whole surface of a cylinder in inches, whose length is 15 feet, and diameter 30 inches?

7. What would be the total pressure on a cylinder, similar in size to the one in the last example, at the rate of 120 lb. to the square inch?

8. What is the whole surface of a cylinder in yards, whose length is 30 feet, and diameter 9 yards?

9. What is the entire surface of a square prism, whose altitude is 15 feet, and each side of the base $11\frac{1}{4}$ feet?

To find the volume or contents of any prism or cylinder.

RULE.

Multiply the area of the base by the altitude.

1. What is the volume of a triangular prism, each side of the base being 4 ft., and the altitude 9 ft.?

ANALYSIS.—The area of the triangle which forms the base of the prism $= \sqrt{48}$, and the volume of the prism $= \sqrt{48} \times 9 =$ 62.352 cubic feet, volume.

2. Find the contents of a cylinder, the diameter of whose base is 2 ft., and altitude 12 feet.

ANALYSIS.—The area of the circle which forms the base of the cylinder $= 2^2 \times .7854 = 3.1416$; $3.1416 \times 12 = 37.6992$ cu. ft., contents.

3. What is the value of a log of walnut, whose cylindrical ends are 3 ft. in diameter, the length being 20 ft., if the walnut is worth 25 cents per cubic foot?

4. What is the weight of a column of marble which measures 4 ft. in diameter and 28 ft. in height, the shape being a right cylinder?

5. An iron casting, in the shape of a parallelopipedon, measures 8 inches in thickness and 15 feet in width, the length being 20 feet; what is its weight?

6. What is the weight of a block of granite which is 5 feet 4 inches long, 2 feet wide, and 3 feet 2 inches thick?

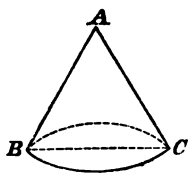
7. A water-pipe measures 3 feet in diameter and 12 feet in length; how many gallons will it hold?

8. What is the volume of a prism, whose base is a square, and its altitude 25 feet 6 inches, each side of the base measuring 6 feet 3 inches?

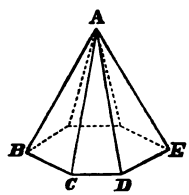
PYRAMIDS AND CONES.

A Pyramid is a solid whose base is a polygon, and whose sides are triangles meeting in a common point, called the **Vertex.**

A Cone is a solid whose base is a circle, and whose convex surface tapers uniformly to a point, called the vertex.



Cone.



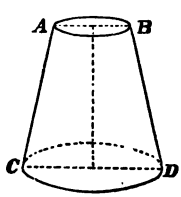
Pyramid.

The **Slant Height** of a pyramid is a line drawn from the vertex to the middle of one of the sides of the base.

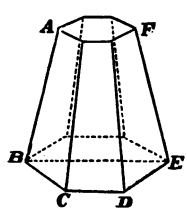
The slant height of a cone is a line drawn from the vertex to the circumference of the base.

The **Altitude** of a pyramid or cone is a line drawn from the vertex perpendicular to the base.

The **Frustum** of a solid is that part which remains after cutting off the upper portion by a plane parallel to the base.



Frustum.



Frustum.

To find the surface of a pyramid or cone.

RULE.

Multiply the perimeter, or the circumference of the base, by half the slant height for the convex surface, and to the product add the area of the base.

1. A stack of hay in the form of a cone is 12 ft. in diameter at the base, and the slant height is 9 ft.; what is the convex surface?

$12 \times 3.1416 \times \frac{9}{2} = 169.6464$ sq. ft., convex surface.

2. What is the convex surface of a pyramid, one side of the square base being 10 ft., and the slant height 8 ft.?

$10 \times 4 \times \frac{8}{2} = 160$ sq. ft., convex surface.

3. What is the cost of painting the outside surface of an octagonal church spire, whose slant height is 75 feet, and one of the equal sides of the base 12 feet, the price being 20 cents per square yard?

4. How many yards of paper, 30 inches in width, will be required to cover the convex surface of a cone 10 feet in diameter at the base, and 18 feet in slant height?

5. How much paper will it require to cover the convex surface of a cylinder 10 ft. in diameter and 18 ft. in height, the width of the paper being 30 inches?

6. Which has the greater convex surface, and how much; a cone 4 feet in circumference, and 4 feet in slant height, or a square pyramid, the perimeter of whose base is 4 feet, and its slant height 4 feet?

7. What is the entire surface of a pyramid, whose slant height is 300 feet, one side of the equilateral triangle which forms its base being 320 feet?

8. Find the whole surface of a cone, the diameter of its base being 30 yards and its slant height 18 feet 10 inches.

To find the volume or contents of a cone or pyramid.

RULE.

Multiply the area of the base by one-third the altitude.

1. What is the solidity of a stick of timber in the form of a cone, the diameter of the base being 6 ft. and the altitude 60 ft.?

$6 \times 6 \times .7854 \times \frac{60}{3} = 565.488$ cu. ft., solidity.

2. A pyramid of brick measures 30 ft. in height; the side of its square base is 20 ft.; what is the weight of the pyramid, and how many bricks does it contain, if a brick weighs 4 lb. and measures $8\frac{1}{2}$ in. $\times$ $4\frac{1}{4}$ in. $\times$ $2\frac{1}{8}$ in.?

3. What is the volume of a triangular pyramid, the altitude being 12 ft., and one side of the equilateral base 4 ft.?

4. A cone measuring 6 ft. in height, and 3 ft. in diameter at the base, is composed of granite; what is its weight?

To find the surface of the frustum of a pyramid or cone.

RULE.

Multiply the sum of the perimeters or of the circumferences of the two ends by half the slant height, for the convex surface; and add the areas of the two ends when the whole surface is required.

1. The slant height of the frustum of a cone is 10 feet, the diameters of the bases being 4 feet and 6 feet, respectively; what is the whole surface?

$(4+6) \times 3.1416 = 31.416$; $31.416 \times \frac{10}{2} = 157.080$ sq. ft., convex surface; $(4^2 + 6^2) \times .7854 = 40.8408$ sq. ft., area of the two ends; $157.08 + 40.8408 = 197.9208$ sq. ft., whole surface.

2. The slant height of the frustum of a square pyramid is 25 feet, the side of one base is 12 feet, and of the other 4 feet; what is the whole surface?

3. The slant height of the surface of the frustum of a cone is 20 feet, the circumferences of the bases being 30 feet and 15 feet respectively; what is the whole surface?

To find the volume of the frustum of a pyramid or cone.

RULE.

To four times the area of a middle section add the areas of the ends, and multiply by one-sixth the altitude.

Note.—By this rule the contents of nearly all regular solids and their frustums can be correctly ascertained. Let pupils practice on the preceding examples in solids, and test its accuracy.

1. The length of a mast is 60 feet, the diameter in the middle is 1 foot, and the diameters of the ends are 6 and 18 inches respectively; what is the number of cubic feet in the mast?

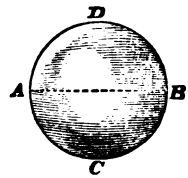
ANALYSIS.—The area of a middle section is $1^2 \times .7854 = .7854$ ft.; the area of the two ends $= (\frac{1}{2})^2 + (1\frac{1}{2})^2 \times .7854 = 1.9635$ ft.; $.7854 \times 4 = 3.1416$ ft. = four times the area of the middle section; $3.1416 + 1.9635 = 5.1051$; $5.1051 \times \frac{60}{6} = 51.051$ cu. ft.

2. The frustum of a square pyramid of marble is 9 feet in height, the side of the lower base is 12 feet, the side of the upper base is 2 feet, the middle section measures 7 feet on each side; what is the weight of the frustum?

3. A lump of gold in the form of the frustum of a cone measures 8 inches in diameter at one end, and 10 inches at the other, its length being 1 foot; what is the value of the lump, if an ounce is worth $20.69?

THE SPHERE.

A **Sphere** is a solid bounded by a curved surface, every part of which is equally distant from a point within it, called the centre.



The **Diameter** of a sphere is a line which passes through the centre and is terminated by the surface. The diameter is frequently called the **Axis.**

The **Radius** of a sphere is a line drawn from the centre to any part of the surface.

To find the surface of a sphere.

RULE.

Multiply the diameter by the circumference.

1. The diameter of a globe is 10 inches; what is the convex surface?

$10 \times 3.1416 \times 10 = 314.16$ sq. in., convex surface.

2. The circumference of a sphere is 1000 feet; what is the convex surface?

3. The diameter of the earth is about 7960 miles; what is its surface?

4. If the radius of a globe is 25 feet, what is the surface?

To find the volume of a sphere.

RULE.

Multiply the cube of the diameter by .5236.

1. What is the solidity of a globe whose diameter is 100 feet?

$100^3 \times .5236 = 1000000 \times .5236 = 523600$ cubic feet, volume.

2. What is the solidity of the earth, if its diameter is 7960 miles?

3. The radius of a ball is 14 inches; what will it cost to gild it, at the rate of 20 cents a foot, and what will it weigh if it is made of walnut?

4. What is the weight of a cannon-ball of cast iron, 12 inches in diameter?

REVIEW PROBLEMS.

1. What is the height of a room whose shape is that of a cube, the solid contents being 3375 cubic feet?

ANALYSIS.—As the solidity is the product of three equal factors, the cube root of 3375 ft., or 15 ft., is the height required.

2. A cubical bin holds 500 bushels; what is the length of one of its inside edges?

3. A mill hopper in the form of the frustum of a square pyramid measures 3 in. on each side at the bottom, and 4 feet at the top, its height being 18 inches; how many bushels of wheat will it hold?

4. If a cubic foot of air at the level of the sea weighs .0768 lb., what is the weight of air which covers one mile square of the ocean, the thickness being reckoned as 1 foot?

5. What is the difference in the weight of two vessels in the form of cylinders, one filled with milk and the other with water, each measuring 1 ft. in diameter and 10 in. in height, the empty vessels weighing 1 pound each?

6. How much more will an iron cannon-ball 10 inches in diameter weigh, than one 5 inches in diameter?

7. How many spheres of gold 1 inch in diameter are equal in weight to a sphere of the same metal, 6 inches in diameter?

8. The area of the base of a cylinder that will contain 1000 gallons, is 10 sq. ft.; what is the altitude?

9. The altitude of a pyramid is 9 ft. and the volume is 192 cu. ft.; what is the side of the square base?

10. The volume of a cone is 36 cu. in.; the area of the base is 6 sq. in.; what is the altitude?

11. The surface of a square prism, excluding the bases, is 144 sq. ft.; the solidity of the prism is 216 cu. ft.; what is the length of one of its sides?

12. The dimensions of a coal-bin are 16 ft. by 8 ft. at the base, and 28 ft. by 10 ft. at the top; the altitude is 6 ft.; how many tons of 2240 lb. each, will the bin hold?

13. What will be the cost of digging a cellar 30 ft. long, 12 ft. wide, and 6 ft. deep, a cubic yard being called a load, and the cost of each load being 50 cents?

GAUGING.

Gauging is the process of finding the capacity of a cask, barrel, or other vessel.

The usual method of obtaining the capacity of a barrel is to reduce its dimensions to those of an equivalent cylinder by finding its *mean diameter*, and apply the rule for the cylinder.

The best method of obtaining the contents of any cask or barrel is by weight; the weight of the empty barrel being marked upon it, and the weight of a cubic foot of the liquid being known.

To find the mean diameter of a cask.

RULE.

Add to the head diameter .5, .55, .6, .65, or .7 of the difference between the bung and head diameters, according to the curvature of the cask. Thus, if the difference of the diameters is but 1 or 2 inches, add .5 of the difference; if the difference is 3 or 4 inches, add .55, and so on.

To find the contents of a cask in gallons.

RULE.

Multiply the square of the mean diameter in inches by the length in inches, and this product by .0034.

Note.—The multiplier .0034 is the quotient obtained by dividing the factor .7854 by 231, the number of cubic inches in a gallon.

1. How many gallons in a cask whose head diameter is 24 inches, bung diameter 26 inches, and length 40 inches?

ANALYSIS.—$26 - 24 = 2$. $24 + (2 \times .5) = 25$ inches, the mean diameter of a cylinder equal in area to the cask. $25^2 \times 40 \times .0034 = 85$ gal.; or, $25^2 \times .7854 \times 40 = 19635$, the area of the cylinder, in inches, and $19635 \div 231$, the number of inches in a gallon, gives 85 gallons.

2. What is the number of gallons in a barrel of vinegar, the weight of the empty barrel being 55 pounds, and the entire weight 269 pounds, a gallon of vinegar weighing $8\frac{3}{4}$ pounds?

ANALYSIS.—269 lb.$-$55 lb.$=$214 lb., the weight of the vinegar; $214 \div 8\frac{3}{4} = 24\frac{16}{35}$, the number of gallons required.

3. Find the contents of a cask of common alcohol by both the preceding methods, the head diameter being 28 inches, the bung diameter 34 inches, and the length 40 inches; the weight of the empty cask being 68 lb., and the entire weight 1006.25 lb.

4. A barrel of pure alcohol weighs 258.43 lb.; the empty barrel weighs 50 lb.; what is the number of gallons in the barrel?

5. A barrel of linseed oil weighs 290.62 lb.; the empty barrel weighs 55 lb.; what is the number of gallons in the barrel?

BOARD MEASURE.

In measuring sawed timber, all estimates are made in *board feet.* A **Board Foot** is 1 foot long, 1 foot wide, and 1 inch thick. Round timber is generally estimated in cubic feet. Hewn timber may be estimated either by cubic or by board measure.

Board feet are changed to cubic feet by dividing by 12, and cubic feet to board feet by multiplying by 12.

Mechanics make use of *duodecimals* in the measurement of lumber.

Duodecimals are compound quantities, whose denominations vary in such a manner that 12 units of any lower denomination make 1 of the next higher.

The denominations are *feet, inches* or *primes, seconds, thirds,* etc.

TABLE.

12 thirds (‴)	make 1 second	= (1″)
12 seconds	make 1 inch or prime	= (1′)
12 inches or primes	make 1 foot	= (1 ft.)

Note.—The product of any two duodecimal quantities, takes a sign equal to the sum of the signs of both factors. Thus, $1'' \times 1'' = 1''''$, or $(\frac{1}{12})^2 \times (\frac{1}{12})^2 = (\frac{1}{12})^4$; as in decimals $.1^2 \times .1^2 = .1^4$ or .0001.

To find the contents of sawed or of hewn timber.

1. What is the number of board feet in a board 20 feet 6 inches long, 9 inches wide, and 1 inch thick?

20 ft. 6′
9′
15 sq. ft. 4′ 6″

ANALYSIS.—We first multiply the 6′ in the multiplicand by the 9′ of the multiplier; $6' \times 9' = 54'' = 4'$ and 6″. Writing the 6″ as a part of the product, we reserve the 4′ to add to the next product, $20 \text{ ft.} \times 9' = 180'$; and $180' + 4' = 184' = 15$ sq. ft. and 4′, which we write as the remainder of the product; 15 sq. ft. 4′ 6″ = 15 sq. ft. 54 sq. in.

RULE.

Write the multiplier under the multiplicand so that units of the same order shall stand in the same column.

Multiply as in compound numbers, carrying a unit for every twelve from each lower to the next higher denomination.

2. How many cubic feet in a stick of timber whose length is 30 ft. 6 in., width 13 in., and thickness 14 in.?

30 ft.	6′	
	13′	
33 sq. ft.	0′	6″
	14′	
38 cu. ft.	6′	7″

ANALYSIS.—We proceed as directed in the rule, and multiply by the 13′, obtaining 33 sq. ft. 0′ 6″ as the board measure of one surface of the stick. Multiplying this surface by 14′ we obtain 38 cu. ft. 6′ 7″ as the cubical contents. 38 cu. ft. 6′ 7″ = 38 cu. ft. 948 cu. in. The same result can be obtained by the ordinary rules of mensuration. Thus, $30\frac{1}{2} \times 1\frac{1}{12} \times 1\frac{1}{6} = \frac{61}{2} \times \frac{13}{12} \times \frac{7}{6} = \frac{5551}{144} = 38\frac{79}{144}$ cu. ft. = 38 cu. ft. 948 cu. in.

3. How many board feet in the piece of timber described in the last example?

38 cu. ft. 6′ 7″ × 12 = 462 sq. ft. 7′ or 462 sq. ft. 84 sq. in., board measure.

ANALYSIS.—As boards are estimated at 1 inch in thickness, the $38\frac{79}{144}$ cu. ft. in the piece of timber will furnish 12 times that number of board feet = $462\frac{7}{12}$ sq. ft., or 462 sq. ft. 84 sq. in.

4. How many board feet in a piece of hewn timber which measures 35 ft. 5 in. in length, 1 ft. 6 in. in width, and 1 ft. 8 in. in thickness? How many cubic feet?

5. Paid $28 a thousand feet, board measure, for 25 pieces of hewn timber, each 24 ft. long, 9 in. wide, and 4 in. thick; what was the entire cost?

6. How many board feet in 260 joists, each 30 ft. long, 13 in. wide, and 5 in. thick?

To find the contents of round timber.

RULE.

To four times the area of a middle section add the areas of the ends, and multiply by one-sixth the length. Four-fifths of the contents will be the equivalent value of hewn timber.

Note.—To find the middle dimension, take half the sum of the dimensions at the ends.

1. How many cubic feet of timber in a log, which measures 3 ft. in circumference at one end, 5 ft. at the other, and 4 ft. in the middle, the length being 40 ft.?

ANALYSIS.—$(\overline{4^2 \times 4} + 5^2 + 3^2) \times .07958 \times \frac{40}{6} = 51.99$ cu. ft., contents. 51.99 cu. ft. $\times \frac{4}{5} = 41.59$ cu. ft. of hewn timber.

2. How many cubic feet of hewn timber can be obtained from a log 50 ft. long, 4 ft. in diameter at one end, 7 ft. at the other, and 5 ft. 6 in. in the middle?

3. What is the exact volume of a log 96 ft. in length, and 15 ft. in circumference in the middle, the ends measuring 12 ft. and 18 ft. in circumference, respectively?

4. How many cubic feet of hewn timber will the log last mentioned yield, and how many board feet?

5. A log in the form of a cylinder is sawed into boards one inch in thickness; how many feet of boards will there be, allowing 20% for waste, the log measuring 60 ft. in length and 5 ft. in diameter?

6. A tapering mast measures 9 feet in circumference at the base, and 2 feet in circumference at the top; how many board feet does it contain if the length is 30 feet?

7. How many cubic feet of hewn timber in a section of a tree which measures 30 feet in diameter at one end, and 8 feet in diameter at the other, the length being 90 feet?

BRICK WORK.

To estimate the number of bricks in a wall.

RULE.

To each of the dimensions of a brick add the thickness of the mortar in which it is laid, and find the contents in cubic inches. Divide the contents of the wall in cubic inches by the contents thus found, and the quotient will be the number of bricks required.

1. The thickness of a wall is $8\frac{3}{4}$ inches; the bricks measure $8\frac{1}{4} \times 4\frac{1}{8} \times 2\frac{3}{8}$ in., and the courses of mortar are $\frac{1}{4}$ of an inch in thickness; how many bricks are in the wall, if the height is 28 ft. and the length 30 ft.?

$$\frac{30 \times 28 \times 8\frac{3}{4} \times 1728}{8\frac{1}{2} \times 4\frac{3}{8} \times 2\frac{5}{8} \times 12} = 10842 \text{ bricks}$$

Note.—When a wall is said to be a certain number of bricks in thickness, to obtain the thickness, we add to the width of a brick the thickness of the mortar, and multiply by the number denoting the thickness.

2. How many bricks $8\frac{1}{4} \times 4\frac{1}{8} \times 2\frac{3}{8}$ in. will be required to build a wall 60 ft. long, 35 ft. high, and $17\frac{1}{2}$ in. thick, the mortar being $\frac{1}{4}$ of an inch in thickness?

3. How many bricks $8\frac{1}{2} \times 4\frac{1}{8} \times 2\frac{3}{8}$ in. will be required to build a wall 40 ft. long, 6 ft. high, and $13\frac{1}{8}$ in. thick, the courses of mortar being $\frac{1}{4}$ of an inch in thickness?

4. How many bricks $8\frac{1}{4} \times 4\frac{1}{8} \times 2\frac{3}{8}$ in. were used in laying a pavement 300 ft. long, and 8 ft. wide?

5. The thickness of a wall is $17\frac{1}{2}$ inches; the bricks measure $8\frac{1}{4} \times 4\frac{1}{8} \times 2\frac{3}{8}$ in.; how many bricks in a section of the wall 1 ft. square and $17\frac{1}{2}$ in. thick?

6. How many cubic feet in a wall which contains 15000 *bricks, the size* of a brick being $8\frac{1}{4} \times 4\frac{1}{8} \times 2\frac{3}{8}$ in., and the *thickness* of the mortar $\frac{3}{8}$ in., if the wall is $8\frac{7}{8}$ in. thick?

HAY.

Hay varies so much in weight, according to the manner in which it is packed, and the quality of the grass, that no rule will give exact results.

Baled hay varies from 10 lb. to 25 lb. to the cubic foot. Hay in the mow, if well settled, will weigh from 4 lb. to 5 lb. to the cubic foot. In an old and well-settled stack, it will weigh from 7 lb. to 9 lb. to the cubic foot.

To estimate the weight of hay in a mow or stack.

RULE.

For hay in the mow, multiply the contents in cubic feet by 4 for clover, or by 5 for timothy. In a well-settled stack, multiply the contents by 7 for clover, or by 9 for timothy hay, and the product will be the weight in pounds.

1. How many pounds of clover in a mow $25 \times 20 \times 8$ ft.?

$$25 \times 20 \times 8 \times 4 = 16000 \text{ pounds.}$$

2. How many pounds of timothy hay in a well-settled conical stack, 8 ft. in diameter and 6 ft. in height?

3. How many pounds of clover hay in a stack which is equal in volume to a cube 12 feet on a side?

4. If a bale of hay measures $8 \times 6 \times 4$ ft., what is its weight in pounds, if a cubic foot weighs 19 lb.?

5. How many pounds of hay in a bale which measures $3\frac{1}{2} \times 2\frac{1}{2} \times 1\frac{1}{4}$ ft., if a cubic foot weighs 17 lb.?

6. The dimensions of a hay-mow are 30 feet in length, 15 feet in width, and 12 feet in height; how many tons of clover hay will it contain?

7. How many tons of timothy hay in a conical stack, if the diameter at the base is 12 ft., the diameter at the top 2 ft., and the height 12 ft.?

COAL.

A cubic foot of anthracite coal, before it is prepared for domestic use, will on an average weigh about 93 lb. When broken for the market it will average about 54 lb.

To estimate the weight of coal in any given space.

RULE.

Multiply the contents in cubic feet by 54, for anthracite, or by 50, for bituminous coal, and the product will be the weight in pounds.

1. How many tons of anthracite coal, of 2240 lb. each, can be stored in a bin 28 ft. long, 20 ft. wide, and 4 ft. deep?

ANALYSIS.—$28 \times 20 \times 4 \times 54 \div 2240 = 54$ tons.

2. How many pounds of bituminous coal in a car 30 ft. long and 7 ft. wide, the depth of the coal being 16 in.?

3. How many pounds of anthracite coal can be placed in a cart which measures 6 ft. in length, $4\frac{1}{2}$ ft. in width, and 16 in. in depth?

4. I wish to build a bin in my cellar to hold 8 tons of anthracite coal, 2240 lb. to the ton; I have made the length 12 ft., and the width 10 ft.; what must be the height of the bin?

5. How many pounds of bituminous coal can be stored in a space $50 \times 50 \times 12\frac{1}{2}$ ft.?

6. How many tons of anthracite coal, 2000 lb. to the ton, can be stored in a yard which measures 60 ft. in length, and 30 ft. in width, the depth of the coal being 6 ft.?

7. A dealer purchases 1500 tons of anthracite coal, 2240 *lb. to the ton,* which he wishes to store in an inclosure 100 *ft. long,* and 80 ft. wide; what will be the depth of the coal?

GENERAL REVIEW PROBLEMS.

1. Three brothers divide an estate of $16000 in the proportion of 3, 4, and 5; the expenses of settlement are $120, which are defrayed by them in proportion to their shares; what are the net proceeds of each man's share?

2. A dealer buys coal at $5.20 per long ton, and sells it at $6.00 per ton of 2000 pounds; what % does he gain?

3. A grocer bought 12⅔ doz. eggs, at 12½ cents per dozen; 10 eggs were destroyed by an accident, 6 doz. were sold at 15 cents per dozen, and the remainder at 4% below cost; did he gain or lose by the transaction, and how much?

4. A teamster refused an offer of 10% advance on the price of a horse which he had just purchased; on account of a defect, however, he was obliged to sell the animal for $110, thereby losing 10%; how much was he offered at first?

5. If a stock bought at 8% discount yields 7% on the investment, at what rate of discount must it be bought to yield 10% on the investment?

6. A merchant in Kansas City received a consignment from New York; it included 5 boxes of prints weighing 1465 lb.; 20 barrels of sugar, 7132 lb.; 23 packages of hardware, 13923 lb.; and 2 barrels of molasses, 1100 lb. The rates to that point, per 100 lb., were $1.58 on the prints, 62 cents on the sugar and molasses, and $1.38 on the hardware. The percentage for each road was as follows:— New York to Pittsburgh 33¼%, Pittsburgh to Chicago 31%, Chicago to St. Louis 23¼%, and St. Louis to Kansas City 12½%. What were the earnings of the different roads on each article, and what was the whole amount paid by *the consignee in* Kansas City?

7. A carpenter owing a merchant, worked for him $9\frac{1}{2}$ days; at settlement it required $\frac{8}{9}$ of the carpenter's wages to pay the bill, leaving him a balance of $\$9\frac{1}{2}$; what did he receive per day?

8. What is the weight of a block of granite, the length being 23 feet, the width 12 feet, and the thickness 9 feet?

9. The cost of certain goods is $7.20. They are marked 30% above cost, and the salesman is instructed to take off 10% for cash; what is the cash price?

10. A merchant has a 30-day note discounted, for which he receives $900; what was the face of the note, if discounted at 6%, on the third day after it was drawn?

11. How many perches of stone, 22 cu. ft. to the perch, in the foundation walls of a house, the length being 30 ft., the width 16 ft., the depth 6 ft., and the thickness of the walls 18 in.?

12. How many cubic yards of earth must be removed, in digging a cellar 30 ft. long, 11 ft. 6 in. wide, and 6 ft. deep, and how many loads will it require to remove the loose material, if 2000 pounds are a load?

13. Three steamers trading from New York to the West Indies make the round trip in 16, 18, and 20 days respectively; if they all start from New York at the same time, how many trips will each be required to make before they are again together in that port?

14. A's rate of walking is to B's as 4 is to 5; if they start from the same point and travel in opposite directions, how far apart will they be at the end of $3\frac{1}{2}$ hours, provided A travels $3\frac{1}{5}$ miles per hour?

15. If $7\frac{1}{2}$ yards of oil cloth $1\frac{1}{4}$ yards wide cost $\$9\frac{3}{8}$, what will be the cost of $15\frac{3}{4}$ yards at the same rate, if the width is $1\frac{1}{8}$ yards?

16. A merchant sells dress goods at 37½ cents per yard, and thereby clears 33⅓%; what would his profit have been on 6 pieces of 48 yards each, if he had sold them at 30 cents per yard?

17. Two men receive a sum of money which they agree to divide in the ratio of ⅔ to ¾; if the smaller share is $17.28, what is the whole sum?

18. A produce dealer sold three lots of potatoes at $.90, $1.12½, and $1.37½ per bushel respectively; he afterwards invested $748 in potatoes, paying for them at the average rate of those sold; how many bushels did he purchase?

19. A merchant bought goods for $1800 which he marked at 20% above cost, but sold them for 5% less than the marked price; what % did he gain?

20. A firm of three persons divided the profits of a year's business; the first received ⅝, the second ¼, and the third the balance; the first man received $1500 more than the third; what was the amount divided?

21. Sold goods costing $180 at 25% advance, and received in payment a note at 60 days, which I had discounted in bank at 7%; what was my profit?

22. The distances from Baltimore to St. Louis are as follows: Baltimore to Parkersburg 384 miles, Parkersburg to Cincinnati 205 miles, Cincinnati to St. Louis 340 miles. A car load of hams, 50 tierces, weighing 23426 pounds, is carried the entire distance at 42 cents per 100 pounds. What is earned by each road in the proportion of its length?

23. How much will it cost, at 20 cents per square yard, to paint the convex surface of a conical spire whose height is 50 feet, the circumference of the base being 100 feet, and the circumference of the top 4 feet?

24. A, B, and C lose $1500 in trade. They agree to divide the loss in the ratio of 2, 3, and 5; what is the loss of each?

25. If 8 yards of cloth $1\frac{1}{2}$ yards wide are enough for a suit of clothes, how many yards are needed when the cloth is $1\frac{1}{8}$ yards wide?

26. A cubical bin holds 2000 bushels of wheat; what are its inside dimensions?

27. A wholesale merchant sells to a retailer at 40% discount, and the retailer sells to the consumer at $66\frac{2}{3}$% profit; what % of the retail price did the goods cost the wholesale merchant, if his profit is 20%?

28. A stevedore, in unloading a vessel, wishes to float 2000 cubic feet of lignum-vitæ by placing it along with some cedar lumber. How many feet (board measure) will be required, allowing 10% additional for the safety of the persons guiding the raft to the shore?

29. Hay can be pressed until a cubic foot weighs 25 pounds; what will be the thickness of a bale into which a ton of hay has been thus pressed, the length being 5 feet, the width being equal to the thickness?

30. A cask filled with pure alcohol contains 130 gallons; what is the weight of the empty cask, if the entire weight is 940 pounds?

31. The distances from New York to Chicago are as follows: New York to Buffalo 440 miles, Buffalo to Cleveland 182 miles, Cleveland to Chicago 356 miles. A car load of grain, 22000 lb., is carried from Chicago to New York at 35 cents per 100 lb. The grain is damaged on the road to the extent of $39.68; what will be the net *earnings* of each road, the loss and the earnings being *divided in* proportion to the lengths of the roads?

32. A cannon-ball 15 inches in diameter weighs 460 pounds; what is the weight of a similar ball 5 inches in diameter, the weight being as $5^3 : 15^3$?

33. A board with tapering sides measures 30 ft. in length; its width at one end is 18 inches, and at the other end 24 inches, the thickness being $1\frac{1}{4}$ inches; how many board feet does it contain?

34. I am to receive \$5000 out of an estate which will require 18 months for settlement; what is the present value of my legacy, money being worth 6%?

35. A farmer buys a team of horses and a wagon at auction, on 9 months' credit; the amount of his bill is \$420; how much would be required to pay it at the end of 3 months, money at 7%?

36. A mechanic finds that he can save \$12 a month; if he wishes to build a house, how large a sum can he borrow so as to pay the interest with his savings?

37. Two steamers sail westward from the same port, one at the rate of 195.25 miles a day, and the other at the rate of 164.5 miles; in how many days will they be $261\frac{3}{8}$ miles apart?

38. A railroad employee has had his wages twice reduced 10%; he now receives \$2.02½ per day; what did he get before the reduction?

39. A machinist now earning \$2.42 per day has had his wages twice increased 10%; what did he get before the increase?

40. How many square feet of tin will cover a roof 16 feet wide and 30 feet long, allowing 5% for joints and waste?

41. What is the value of £1100 19 s. 6 d., at \$4.8665 *per pound sterling*?

42. A merchant's stock of goods cost him $78965.34. His sales amount to $60543.25, and the goods on hand cost $32575.69. What is his per cent. of profit on the goods sold?

43. Lent a man $5000 for 3 years; at the end of that time he paid me $6350; what was the rate of interest?

44. A merchant buys goods as follows:—

June 1,	1876,	$1500	on	9	months.
July 4,	"	1000	"	7	"
Aug. 1,	"	500	"	6	"
Sept. 6,	"	1100	"	4	"
Oct. 5,	"	800	"	6	"

October 27, he gives a note for the whole amount. What should be the time of the note?

45. If a mechanic invests his money in a building society that earns annually 8 per cent. compound interest, how much will he save in 9 years, his deposits being $120 a year?

46. A railroad company wishes to build a tank that will contain 4000 gallons of water; what must be the depth, if the length is 16 feet and the width 8 feet?

47. A merchant sold goods for $1000, after allowing 5 per cent. for prompt payment; what did the goods cost him, he having gained 20 per cent.?

48. What weight can be sustained by a balloon that weighs 500 lb., the shape being that of a sphere, whose diameter is 60 feet; the balloon being filled with a gas which weighs $\frac{3}{20}$ of an ounce to the cubic foot, air weighing $1\frac{1}{5}$ ounces to the cubic foot?

49. A vessel sails 8 miles an hour in still water; going up stream it requires 72 minutes to sail 8 miles; how long will it require, going down stream?

50. A bankrupt offers to pay a creditor, to whom he owes \$10000, 40% in cash, or guaranteed notes at 3, 6, and 9 months for \$1500 each; which is the better offer, and by how much, money being worth 10%?

51. My house rents for \$420 a year; the taxes and repairs amount to \$75 a year; allowing 6 per cent. per annum for interest, what is the value of the house?

52. How many bricks, 8 inches long, $4\frac{1}{8}$ inches wide, and $2\frac{1}{4}$ inches thick, will be required in building a house 40 feet long, 20 feet wide, and 30 feet high, the walls being $13\frac{1}{8}$ inches thick, and the courses of mortar $\frac{1}{4}$ of an inch thick; the house having 4 doors, each 7 feet high and 46 inches wide, and 18 windows, each 38 inches wide and 70 inches in height?

53. How many feet of boards will it require to lay two floors in the above house, each floor being of the full size of the inside dimensions of the building; the allowance for waste being one-sixth?

54. How many bricks $8\frac{1}{4} \times 4\frac{1}{8} \times 2\frac{3}{8}$ in. will it require for a pavement 20 feet long and 8 feet wide?

55. How many cubic quarter inches are there in one cubic inch?

56. A man working on a railroad earns \$1 a day more than his board, paying 50 cents a day for the latter; at the end of 60 days he has saved \$25; how many days was he idle?

57. Which is the better investment, a 6% stock bought at 108, or a 5% stock at 90, the par value being 100?

58. How many yards of carpet 27 inches wide will it require to cover the floors of a room 26 ft. long, and 11 ft. 3 in. wide; and what will be the cost, at \$2.25 a yard?

59. A house that cost $3800 is rented for $35 a month. The owner pays $65 a year for taxes and $38 for repairs; what per cent. does the house pay on its cost?

60. What sum must be invested in a 10 per cent, stock that costs $134, the par value being $100, to derive an income of $1000 per annum?

61. How many tons of clover hay can be packed in the upper part of a barn, level with the eaves, the rectangular dimensions being 27 feet by 40 feet, and the height 11 feet?

62. How much farther does a man walk in going round two sides of a square field containing 5 acres, than in walking diagonally across the field from opposite corners?

63. What is the equated time of the following account, and the cash balance May 25, 1877, allowing 6% interest?

Dr. PETER SIMMONS. *Cr.*

1877.				1877.			
Apr. 2	To Mdse.	$875	50	Apr. 4	By Cash	$325	00
" 16	" " at 30 days	200	00	" 12	" Mdse.	180	00
" 25	" Cash	500	75	May 1	" Note at 90 days	1000	00
May 9	" Mdse. at 60 days	750	00	" 10	" Mdse.	600	75
" 18	" " " 90 "	250	00	" 17	" Cash	50	00
" 25	" " " 30 "	1000	00	" 25	" "	1200	00

64. William Thompson, John Henry, and Joseph Wilkins formed a partnership with a capital of $20000; Thompson's share of the capital is $3000, Henry's share is $15000, and Wilkins contributes $2000. At the end of 3 months Wilkins contributes $5000 to supply the place of an equal amount withdrawn by Henry. Thompson closes his connection with the firm and receives his $3000 at the end of 10 months. At the end of a year they divide $6000. What is the amount received by each, allowing Thompson his fair share of the gains?

ALLIGATION.

Alligation treats of the mixture of articles of different values to form a compound of a certain value.

CASE I.

To find the average price of a mixture, the quantity and cost of each article being given.

1. A grocer mixes 5 pounds of sugar worth 10 ct. a pound, with 16 pounds worth 12 ct. a pound, and 25 pounds worth 16 ct. a pound; what is the value of one pound of the mixture?

$5 \times 10 = 50$
$16 \times 12 = 192$
$25 \times 16 = 400$
$46 \quad)642(13\frac{24}{46}$ ct.

ANALYSIS.—The 46 pounds of sugar are worth 642 cents, and 1 pound is worth $\frac{1}{46}$ of 642 ct., or $13\frac{24}{46}$ ct.

RULE.

Find the total value of all the articles, and divide by the number which denotes the sum of the articles.

2. A miller mixes 20 bushels of wheat worth $1.00 per bushel with 25 bushels worth $1.10 and 30 bushels worth $1.25; what is the value of a bushel of the mixture?

3. If I mix 26 gallons of oil worth 75 ct. a gallon, 32 gallons worth 80 ct. a gallon, 50 gallons worth 40 ct. a gallon, and 150 gallons worth 90 ct. a gallon, what is the value of a single gallon of the mixture?

4. By accident some cigars were mixed, worth 8, 10, and 12 dollars per hundred respectively, there being 200 of the first, 100 of the second, and 200 of the third; how should they be sold apiece to gain 20 per cent. on the cost?

CASE II.

To find the quantity to be used of each article, when the average cost and the cost of each article are given.

1. A grocer wishes to mix sugar at 10 ct. a pound with some at 16 ct. a pound, so that a pound of the mixture shall be worth 11 ct.; how much of each must he take?

$$11 \left\{ \begin{matrix} 10 \\ 16 \end{matrix} \right. \begin{matrix} \text{5 pounds of 10-cent sugar.} \\ \text{1 pound of 16-cent sugar.} \end{matrix}$$

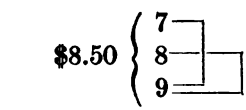
ANALYSIS.—It is evident that if the grocer puts in 1 pound of the 16-cent sugar, and sells it for 11 cents, he loses 5 cents, and as on each pound of 10-cent sugar he gains 1 cent, it will be necessary to mix 5 pounds of the 10-cent sugar in order that the gain on it shall exactly balance the loss on 1 pound of 16-cent sugar. He must therefore use 5 pounds of the 10-cent sugar to 1 pound of the 16-cent sugar.

Note.—Analysis is a simple and philosophical method of solving examples in Alligation, and the pupils should be encouraged to solve all such problems by this method.

RULE.

Write the average cost by itself, and the cost of each article in a column on the right. Link each value that is less than the average cost with one that is greater. Place the difference between a less value and the average cost, opposite the greater value with which such less value is linked, and the difference between a greater value and the average cost, opposite the less value with which it is linked. If there is only one difference opposite to any value, it will be the required quantity of the article of that value; but if there should be two or more differences, their sum will express the required quantity.

2. What number of barrels of flour worth \$7, \$8, and \$9 per barrel must be sold, to realize \$8.50 as an average price per barrel?



Note.—Following the directions contained in the rule, we find that ½ barrel of \$7 flour, ½ barrel of \$8 flour, and 2 barrels of \$9 flour will fulfil the conditions of the question. If we wish for any greater number of barrels, we can multiply each of the three quantities by the same number and still preserve the proportion. Thus, multiplying by 10, we shall have 5 barrels of \$7, 5 of \$8, and 20 of \$9 flour.

3. In what quantities must teas be mixed, worth 75 ct., \$1.00, \$1.25, and \$1.50 per pound, respectively, in order that a pound of the mixture shall be worth 85 cents?

4. How shall three grades of coffee be mixed, the first costing 20 ct. per pound, the second 24 ct., and the third 40 ct. so that a pound of the mixture can be sold for 30 ct. and a profit of 25% be made on the transaction?

5. I have apples worth 25 ct. a dozen, oranges worth 50 ct. a dozen, and lemons worth 40 ct. a dozen; how many dozen of each must I sell that the average price may be 45 cents?

6. Bought four kinds of cloth, at 75 ct., \$1.25, \$2, and \$3 per yard; how many yards did I buy of each, if the average cost is \$2.50 per yard?

7. How much water must be mixed with milk worth 10 ct. a quart, that it may be sold at 8 ct. a quart without loss to the seller?

8. How shall three qualities of gold, 12 carats, 20 carats, and 22 carats fine, respectively, be mixed so that the mixture shall be 18 carats in fineness?

CASE III.

When one of the articles is limited in quantity.

1. How shall 20 pounds of coffee, worth 20 ct. a pound, be mixed with coffee worth 24 ct. and coffee worth 40 ct. a pound, so that the mixture shall be worth 30 ct. a pound?

BY CASE II.

$$30 \left\{ \begin{array}{l} 20 \\ 24 \\ 40 \end{array} \right. \begin{array}{l} 10 \\ 10 \\ 10 + 6 = 16 \end{array} \quad \begin{array}{l} 10 \times \frac{2}{10} = 20 \text{ lb. of } 24 \text{ ct.} \\ 16 \times \frac{2}{10} = 32 \text{ " } 40 \text{ "} \end{array}$$

ANALYSIS.—We find by Case II, without regard to the quantity of the 20-cent coffee, that the proportional parts are 10 lb. of 20-cent, 10 lb. of 24-cent, and 16 lb. of 40-cent coffee. But as we have 20 lb. of 20-cent coffee we must take $\frac{2}{10}$, or 2 times as much of each of the other kinds, or 20 lb. of 24-cent and 32 lb. of 40-cent coffee.

RULE.

Find the proportional quantities of each article by Case II. Divide the given quantity by the proportional quantity of that article, and multiply the remaining proportional quantities by the quotient.

2. A wine merchant wishes to mix three grades of wine, worth \$1, \$2, and \$3 per gallon, so as to sell the mixture at \$2.75 per gal. Having but 20 gallons of the first, how much will he require of each of the others?

3. How much water must be mixed with 20 gallons of alcohol, worth \$2 per gallon, so that a gallon of the mixture can be sold for \$1.50?

4. Bought turkeys at \$1 apiece, geese at 75 cents, ducks at 50 cents, and 15 chickens, at 30 cents each; how many were there of each, if the average cost was 60 cents?

5. How much sugar worth 12 cents a pound must be mixed with 10 pounds valued at 10 cents a pound, so that the mixture shall be worth $10\frac{1}{2}$ cents per pound?

CASE IV.

When two or more of the articles are limited in quantity.

1. How much gold, 10, 12, and 14 carats fine, must be mixed with 2 ounces of 20 carats, and 3 ounces of 22 carats fine, in order that the mixture shall be 18 carats fine?

BY CASE I.

$2 \times 20 = 40$
$3 \times 22 = 66$
$5 \quad)106$
$21\frac{1}{5}$

BY CASE II.

18 { 10, 12, 14, $21\frac{1}{5}$ }

$3\frac{1}{5}$
$3\frac{1}{5}$
$3\frac{1}{5}$
$8 + 6 + 4 = 18.$

BY CASE III.

$3\frac{1}{5} \times \frac{5}{18} = \frac{8}{9}$ oz. of 10 carats fine.
$3\frac{1}{5} \times \frac{5}{18} = \frac{8}{9}$ oz. of 12 "
$3\frac{1}{5} \times \frac{5}{18} = \frac{8}{9}$ oz. of 14 "

ANALYSIS.—We first find by Case I the average fineness of the two quantities given, which is $21\frac{1}{5}$ carats for the 5 ounces; then by Case II we find the proportional quantities required, to be $3\frac{1}{5}$ ounces each of the 10, 12, and 14 carats to 18 ounces of the mixture of 20 and 22 carats; but as we have only 5 ounces of the mixture we will require (Case III) $\frac{5}{18}$ of $3\frac{1}{5}$ ounces $= \frac{8}{9}$ oz. each of the 10, 12, and 14 carats.

RULE.

Find the average value of articles that are limited in quantity, by Case I; the proportional quantities required, by Case II; and the amount of each article not limited in quantity, by Case III.

2. A grocer desires to mix coffee worth 16 cents a pound, with 15 pounds of an article worth 25 cents per pound, and 15 pounds worth 35 cents per pound, so that the mixture can be sold for 18 cents per pound, without loss; how many pounds of the first will there be in the mixture?

3. How much gold 15 carats fine must be added to 14 ounces 20 carats fine, and 2 ounces 21 carats fine, so that the mixture shall be 16 carats fine?

4. Bought flour at $4 and at $6 per barrel, and have on hand 20 barrels worth $5, and 20 barrels worth $10 per barrel; how many barrels did I buy if the average cost of all is $7?

5. A merchant has cloth worth $2 a yard, and cloth worth $4 a yard, and buys 50 yards at $5 a yard, and 100 yards at $3.50 a yard; how much has he of each of the first two, if the average cost of all is $3?

CASE V.

When the whole compound is limited to a particular quantity.

1. A grocer wishes to mix tea at 50 cents, 75 cents, and 80 cents a pound; how many pounds of each must there be in 100 pounds of the mixture, the average price being 60 cents a pound?

BY CASE II.

$$60 \left\{ \begin{array}{l} 50 \\ 75 \\ 80 \end{array} \right. \begin{array}{l} 15 + 20 = 35 \\ 10 \\ 10 \end{array}$$

$35 + 10 + 10 = 55.$

$35 \times \frac{100}{55} = 63\frac{7}{11}$ lb. of 50 ct.

$10 \times \frac{100}{55} = 18\frac{2}{11}$ " 75 "

$10 \times \frac{100}{55} = 18\frac{2}{11}$ " 80 "

ANALYSIS.—By Case II we find the proportional quantities to be 35 lb. of 50 ct. tea, and 10 lb. each of 75 and 80 ct. tea. Adding these amounts, we have 55 pounds; but we need 100 pounds: hence we must have $\frac{100}{55}$ times the proportional quantity of each article.

RULE.

Find the proportional quantities by Case II, and divide the given quantity by their sum. Multiply each of the proportional quantities by the quotient thus obtained, and the products will be the required quantities.

2. How much water must be mixed with cider worth 20 cents a gallon, in order that 40 gallons of the mixture shall be worth 15 cents a gallon?

BY CASE II.

$$15\left\{\begin{matrix}0\\20\end{matrix}\right]\begin{matrix}5\\15\end{matrix}$$

$5 \times \frac{40}{20} = 10$ gal. of water.
$15 \times \frac{40}{20} = 30$ " cider.

3. A farmer has in his barn oats worth 30 cents a bushel, wheat worth 90 cents a bushel, and rye worth 50 cents a bushel; how many bushels has he of each kind, if the average price of the whole is 54 cents, there being 500 bushels in the barn?

4. Sugars worth 8, 9, 10, 11, and 12 cents a pound, are to be mixed, so that 200 pounds of the mixture shall be worth $10\frac{1}{2}$ cents a pound; how much of each sort must be taken?

5. 100 pounds of gold-bearing quartz were bought for $100. There were three kinds, costing $4 a pound, 25 cents a pound, and 50 cents a pound respectively; how many pounds were there of each kind?

6. How shall I mix gold that is 10 carats fine, with gold 14 carats, and 18 carats fine, respectively, to obtain 6 ounces of a mixture 16 carats fine?

HORNER'S METHOD OF EVOLUTION.

1. Extract the square root of 625.

0	625(25
2	4
2	225
2	225
40 Trial Divisor.	
5	
45 Complete Divisor.	

ANALYSIS.—The greatest square contained in the left-hand period is 4, the root of which is 2; this we write as the first figure of the required root. We next add the figure thus found to the cipher at the head of the first left-hand column, and obtain 2, which being multiplied by the 2 in the root gives 4. We write the 4 under the first left-hand period of the given number, and subtract it, leaving 2 as a remainder. We next add the quotient figure 2 to the 2 in the first column, obtaining 4, to which we annex one cipher for a trial divisor, and, bringing down the next period 25, we have 225 as the new dividend. We find that the trial divisor 40 is contained in this dividend 5 times. We next add the 5 thus found to the last term of the first column, completing the divisor. Multiplying this result by the quotient figure 5 gives 225, which we subtract from the dividend, and, as there is no remainder, 25 is the exact square root of 625.

RULE.

Prepare as many columns as there are units in the index of the required root, placing the given number at the top of the first right-hand column, and a cipher at the head of each of the others.

Separate the given number into periods of as many figures each as there are units in the index of the required root, and find by trial the first figure of the root.

Add the figure thus found to the cipher at the head of the first left-hand column, multiply the sum by the root figure, and add the product to the cipher at the head of the second column. Proceed in a similar manner until the last product

falls under the first left-hand period of the given number. Subtract this product from this first left-hand period, and to the remainder annex the next period for a new dividend.

Add the figure of the root just found to the amount in the first column, multiply the sum by the same figure, adding the product to the next column, and so continue till the last product falls in the last column but one.

Proceed in a similar manner, stopping each time one column farther to the left, until the last operation is the addition of the first figure of the root, to the sum in the first left-hand column.

Annex one cipher to the number at the foot of the first column, two ciphers to the number at the foot of the second column, and so on.

Consider the number standing in the column next to the last as the trial divisor, find how often it is contained in the dividend, and write the quotient as the second figure of the root.

Add this figure to the amount in the first column, multiply the sum by the same figure, and proceed as before, till the last product falls under the trial divisor. The sum of the last product and the trial divisor will be the complete divisor. Multiply by the figure of the root just found, subtract the product from the dividend, and proceed as before till all the periods have been brought down.

Note 1.—When any dividend will not contain the trial divisor, place a cipher in the root, bring down the next period, annex one cipher to the number in the first column, two ciphers to the number in the second column, and so on, using the trial divisor with the proper number of ciphers annexed as a new trial divisor.

Note 2.—When the exact root cannot be extracted, periods of ciphers may be annexed, and the work extended to any required number of decimal places.

2. Extract the cube root of 1728.

0	0	1728(12
1	1	1
1	1	)728
1	2	728
2	300	Trial Divisor.
1	64	
30	364	Complete Divisor.
2		
32		

PROOF.—$12 \times 12 \times 12 = 1728$.

3. What is the fourth root of 390625?

0	0	0	390625(25
2	4	8	16
2	4	8	)230625
2	8	24	230625
4	12	32000	Trial Divisor.
2	12	14125	
6	2400	46125	Complete Divisor.
2	425		
80	2825		
5			
85			

PROOF.—$25 \times 25 \times 25 \times 25 = 390625$.

4. What is the square root of 2.5 to three decimal places?

5. What is the cube root of 2.7?

6. Find the value of $\sqrt[5]{.5}$.

7. What is the fourth root of 4?

8. Find the value of $\sqrt[3]{5^6}$, or, the cube root of the sixth power of 5.

9. Extract the third root of 3 to three decimal places.

10. Find the difference between the square root of the cube of 3 and the cube root of the square of 3, or $\sqrt{3^3} - \sqrt[3]{3^2}$.

11. Extract the third root of .01.

12. What is the value of $\sqrt[4]{.2}$?

13. Find the square root of the square root of .2, to two decimal places, or $\sqrt{\sqrt{.2}}$.

14. What is the value of $\sqrt[4]{\sqrt{6561}}$?

15. Extract the cube root of 523606616.

16. What is the value of $\sqrt[4]{50625}$?

17. What is the value of $\sqrt[5]{1048576}$?

18. What is the fifth root of 1889568?

19. Find the third root of 32.768.

20. What is the fourth root of 1099511627776?

21. What is the sixth root of 5 to one decimal place?

ARITHMETICAL PROGRESSION.

An **Arithmetical Progression** is a series of numbers either increasing or decreasing by a common difference.

Thus: 1, 3, 5, 7, 9 is an increasing series.

9, 7, 5, 3, 1 is a decreasing series.

The common difference in each series is 2.

The numbers which form the series are called the *Terms.* The first term and the last term are called the *Extremes,* and the other terms, the *Means.*

The following five quantities are considered in Arithmetical Progression: The *first term;* the *last term;* the *common difference;* the *number of terms;* and the *sum of the series.*

CASE I.

One of the extremes, the common difference, and the number of terms being given, to find the other extreme.

1. The first term of an increasing series is 2, the common difference 3, and the number of terms 10; what is the last term?

$10 - 1 = 9$; $(9 \times 3) + 2 = 29$, the last term.

ANALYSIS.—As there are ten terms, we have 9 differences of 3, or $9 \times 3 = 27$; the last term is therefore equal to 27 more than the first, or $27 + 2 = 29$.

2. The first term of a decreasing series is 29, the common difference is 2, and the number of terms 10; what is the last term?

$10 - 1 = 9$; $29 - (9 \times 3) = 2$, the last term.

ANALYSIS.—As there are 9 differences of 3, or $9 \times 3 = 27$, the last term is therefore equal to 27 less than the first, or $29 - 27 = 2$.

RULE.

To the less extreme add the product of the common difference by the number of terms less one; or, subtract this product from the greater extreme; the result in either case will be the other extreme.

3. The first term of an increasing series is 4, the common difference 5, and the number of terms 30; what is the last term?

4. The last term of a decreasing series is 300, the common difference 5, and the number of terms 40; what is the first term?

5. The first term of an increasing series is 25, the common difference 25, and the number of terms 25; what is the last term?

6. The last term of an increasing series is $19\frac{1}{2}$, the common difference is $\frac{1}{2}$, and the number of terms is 20; what is the first term?

7. An arithmetical progression has $30\frac{1}{4}$ for the first term of a decreasing series, the common difference is $\frac{3}{4}$, and the number of terms is 36; what is the last term?

CASE II.

To find the common difference, when the extremes and the number of terms are given.

1. The extremes of a series are 2 and 10, and the number of terms is 5; what is the common difference?

$10 - 2 = 8$, $8 \div 4 = 2$, the common difference.

ANALYSIS.—It has been shown in Case I, that the difference between the extremes is equal to the number of terms less one multiplied by the common difference; from this it follows that the difference of the extremes divided by the number of terms less one, or $8 \div 4 = 2$, is the common difference.

RULE.

Divide the differences between the extremes, by the number of terms less one.

2. The extremes of a series are 1 and 50, and the number of terms is 8; what is the common difference?

3. The first term is 9, the last term 54, and the number of terms 6; what is the common difference?

4. If \$100 will amount in 10 years to \$200, what is the rate per cent., simple interest?

5. The first term is $\frac{1}{4}$, the last term $10\frac{1}{4}$, and the number of terms 41; what is the common difference?

6. A merchant discharged a debt in 6 monthly payments; the first time he paid \$70, and the last time \$170; by what amount did he increase his payments each time?

7. 10 pieces of cloth were sold at prices increasing by a common difference; the first sold at $1.25 a yard, the last sold at $2.15 a yard; what was the common difference?

CASE III.

To find the number of terms, when the extremes and the common difference are given.

1. The extremes are 5 and 20, and the common difference is 5; what is the number of terms?

$20 - 5 = 15$, $15 \div 5 = 3$, $3 + 1 = 4$, the number of terms.

ANALYSIS.—As the difference between the extremes is equal to the common difference multiplied by the number of terms less one, it follows that by dividing the difference between the extremes, 15, by the common difference, 3, and adding 1 to the quotient, we obtain 4 as the number of terms.

RULE.

Divide the difference between the extremes by the common difference, and add 1 to the quotient.

2. The extremes are 10 and 100, and the common difference is 10; what is the number of terms?

3. The extremes are $2\frac{1}{4}$ and 9, and the common difference is $\frac{3}{4}$; what is the number of terms?

4. If a body falls by the force of gravity $16\frac{1}{12}$ feet in the first second, and gains $32\frac{1}{6}$ feet each succeeding second, how many seconds has a cannon-ball been falling, if it has passed through $209\frac{1}{12}$ feet during the last second of its fall?

5. In what number of years will $100 amount to $200, at 10 per cent., simple interest?

6. The first term is 200, the last term 100, and the common difference 10; what is the number of terms?

CASE IV.

To find the sum of the series, when the extremes and the number of terms are given.

1. The extremes are 2 and 10, and the number of terms is 5; what is the sum of the series?

$$\frac{(10+2)}{2} \times 5 = 30, \text{ the sum of the series.}$$

$$\begin{array}{r} 2+\ 4+\ 6+\ 8+10=30 \\ 10+\ 8+\ 6+\ 4+\ 2=30 \\ \hline 12+12+12+12+12=60 \end{array}$$

ANALYSIS.—The common difference is found to be 2. Writing the series in full and then in reverse order, we take the sum of each pair of terms, and, adding these sums together, find that the amount 60, which is twice the sum of the series, is equal to 5 times $(10+2)$ the sum of the extremes, and the sum of the series is $\frac{(10+2)\times 5}{2}$ $=30$.

RULE.

Multiply the sum of the extremes by the number of terms, and divide the product by two.

2. The first term is 10, the last term 60, and the number of terms 26; what is the sum of the series?

3. The extremes are 5 and 25, and the number of terms 26; what is the sum of the series?

4. The extremes are $16\frac{1}{12}$ and $209\frac{1}{12}$, and the number of terms is 7; what is the sum of the series?

5. A body falls $16\frac{1}{12}$ feet the first second, gains $32\frac{1}{6}$ feet each second after the first, and falls for 13 seconds; what is its final velocity, and how far does it fall?

6. A number of telegraph poles are to be planted 75 yards apart; how far will a man travel who starts at the first one and plants ten of them, returning once from each pole to the starting point?

GEOMETRICAL PROGRESSION.

A **Geometrical Progression** is a series of numbers, each of which varies from the preceding in a constant ratio.

The numbers which form the series are called the *Terms*. The first term and the last term are called the *Extremes*, and the other terms, the *Means*.

The **Ratio** is the constant multiplier by which the successive terms are obtained.

An **Increasing Series** is one in which the ratio is greater than unity.

A **Decreasing Series** is one in which the ratio is a proper fraction.

An **Infinite Series** is a decreasing series in which the number of terms is infinite.

The following five quantities are considered in geometrical progression: The *first term;* the *last term;* the *ratio;* the *number of terms;* and the *sum of the series.*

CASE I.

One of the extremes, the ratio, and the number of terms being given, to find the other extreme.

1. The first term of a geometrical series is 3, the ratio 2, and the number of terms 12; what is the last term?

$$3 \times 2^{11} = 3 \times 2048 = 6144, \text{ the last term.}$$

ANALYSIS.—The second term will be 3×2, the third term 3×2^2, and the 12th or last term 3×2^{11} or 6144.

RULE.

Multiply the first term by that power of the ratio whose index is one less than the number of terms, and the product will be the last term; or, divide the last term by the same power of the ratio, and the quotient will be the first term.

2. The first term of a geometrical series is 4, the ratio 5, and the number of terms 6; what is the last term?

3. The last term of a series is 531441, the ratio 3, and the number of terms 12; what is the first term?

4. The last term is 63.123848, the ratio 1.06, and the number of terms 5; what is the first term?

5. What is the amount of $80 at compound interest for 5 years, the rate being 6 per cent.?

Note.—The ratio is 1.06, the number of terms $5 + 1 = 6$, and the amount is the last term.

6. What is the amount of $10000 at 5 per cent. compound interest for 10 years?

7. The first term is 10, the ratio $\frac{1}{2}$, and the number of terms 6; what is the last term?

CASE II.

To find the ratio, when the extremes and the number of terms are given.

1. The first term is 4, the last term 2500, and the number of terms 5; what is the ratio?

$$\frac{2500}{4} = 625; \quad \sqrt[4]{625} = 5, \text{ the ratio.}$$

ANALYSIS.—As we multiply the first term 4 by the 4th power of the ratio to obtain 2500 the last term, if we divide 2500 by 4, the quotient 625 is the 4th power of the ratio, and extracting the 4th root of this quotient we obtain 5 as the required ratio.

RULE.

Divide the last term by the first, and extract that root of the quotient whose index is one less than the number of terms.

2. The extremes are 4 and 256, and the number of terms is 4; what is the ratio?

3. The first term is 2, the last term $\frac{1}{16}$, the number of terms 6; what is the ratio?

4. The first term is 25, the last term 31.561924, the number of terms 5; what is the ratio?

5. $50 at compound interest will amount to $63.123848 in 4 years; what is the rate per cent.?

Note.—The number of terms is 5.

CASE III.

To find the number of terms, when the ratio and the extremes are given.

1. The ratio is 5, the extremes are 4 and 2500; what is the number of terms?

$\frac{2500}{4} = 625 = 5^4$; $4 + 1 = 5$, the number of terms.

ANALYSIS.—Dividing the last term by the first, we obtain 625, that power of the ratio which is one less than the number of terms. $625 = 5^4$, and $4 + 1 = 5$, the number of terms required.

RULE.

Divide the last term by the first, raise the ratio to a power equal to this quotient, and add one to the index of that power for the number of terms.

2. The extremes are 3 and 96, and the ratio is 2; what is the number of terms?

3. The extremes are 25 and 31.561924, and the ratio 1.06; what is the number of terms?

4. In what number of years will $100 amount to $157.351936 at 12 per cent. compound interest?

Note.—The number of terms is one more than the required number of years.

5. The extremes are 4 and $\frac{1}{2}$, and the ratio is $\frac{1}{2}$; what is the number of terms?

CASE IV.

To find the sum of the series, when the ratio and the extremes are given.

1. The extremes are 2 and 54, and the ratio is 3; what is the sum of the series?

$$\frac{(54 \times 3) - 2}{3 - 1} = 80, \text{ the sum of the series.}$$

$$\begin{aligned} 6 + 18 + 54 + 162 &= 240 \\ 2 + 6 + 18 + 54 &= 80 \\ \hline -2 + 0 + 00 + 00 + 162 &= 160 \end{aligned}$$

ANALYSIS.—Multiplying each term of the given series by the ratio 3, we obtain a series the sum of which is 3 times as great as the sum of the given series. Subtracting the latter series term by term from the former, there remains the difference between the first term of the one and the last term of the other, or $(54 \times 3) - 2$, which is evidently equal to twice the sum of the given series, and the sum of the given series is $\frac{(54 \times 3) - 2}{3 - 1} = 80$.

RULE.

Multiply the last term by the ratio, and divide the difference between this product and the first term by the difference between the ratio and one.

2. The extremes are 3 and 648, and the ratio is 6; what is the sum of the series?

3. The extremes are 2 and $\frac{1}{32}$, and the ratio is $\frac{1}{2}$; what is the sum of the series?

4. What is the sum of the infinite series $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$, etc.?

Note.—The last term may be taken as 0.

5. The extremes are \$5 and \$1280, and the ratio 4; what is the sum of the series?

ANNUITIES.

An **Annuity** is a sum of money which is payable at regular periods of time.

A **Certain Annuity** is one that continues for a fixed period of time.

A **Perpetual Annuity,** or **Perpetuity,** is one that continues forever.

A **Contingent Annuity** is one that begins or ends, or begins and ends on the occurrence of certain specified future events, as on the death of one or more persons.

An **Annuity in Reversion** is one that begins at a specified future time, or on the occurrence of a specified future event.

An **Annuity in Arrears,** or **Forborne,** is one the payments of which have been allowed to accumulate, instead of being paid when due.

The **Amount** of an annuity is the sum of all the payments plus the interest on each payment from the time it becomes due until the final payment.

The **Present Worth** of an annuity is that sum of money which will, in the given number of payments and at the given rate, amount to the final value.

Pensions and ground-rents are of the nature of annuities. *Annuities* are usually computed at compound interest.

TABLE.

The amount of an annuity of $1 for any number of years from 1 to 25.

YR.	2%	3%	4%	5%	6%
1	$1.000000	$1.000000	$1.000000	$1.000000	$1.000000
2	2.020000	2.030000	2.040000	2.050000	2.060000
3	3.060400	3.090900	3.121600	3.152500	3.183600
4	4.121608	4.183627	4.246464	4.310125	4.374616
5	5.204040	5.309136	5.416323	5.525631	5.637093
6	6.308121	6.468410	6.632975	6.801913	6.975319
7	7.434283	7.662462	7.898294	8.142008	8.393838
8	8.582969	8.892336	9.214226	9.549109	9.897468
9	9.754628	10.159106	10.582795	11.026564	11.491316
10	10.949721	11.463879	12.006107	12.577893	13.180795
11	12.168715	12.807796	13.486351	14.206787	14.971643
12	13.412090	14.192030	15.025805	15.917127	16.869941
13	14.680332	15.617790	16.626838	17.712983	18.882138
14	15.973938	17.086324	18.291911	19.598632	21.015066
15	17.293417	18.598914	20.023588	21.578564	23.275970
16	18.639285	20.156881	21.824531	23.657492	25.672528
17	20.012071	21.761588	23.697512	25.840366	28.212880
18	21.412312	23.414435	25.645413	28.132385	30.905653
19	22.840559	25.116868	27.671229	30.539004	33.759992
20	24.297370	26.870374	29.778079	33.065954	36.785591
21	25.783317	28.676486	31.969202	35.719252	39.992727
22	27.298984	30.536780	34.247970	38.505214	43.392290
23	28.844963	32.452884	36.617889	41.430475	46.995828
24	30.421862	34.426470	39.082604	44.501999	50.815577
25	32.030300	36.459264	41.645908	47.727099	54.864512

TABLE.

The present worth of an annuity of $1 for any number of years from 1 to 25.

YR.	2%	3%	4%	5%	6%
1	$ .980392	$ .970874	$ .961538	$ .952381	$ .943396
2	1.941561	1.913470	1.886095	1.859410	1.833393
3	2.883883	2.828611	2.775091	2.723248	2.673012
4	3.807729	3.717098	3.629895	3.545951	3.465106
5	4.713460	4.579707	4.451822	4.329477	4.212364
6	5.601431	5.417191	5.242137	5.075692	4.917324
7	6.471991	6.230283	6.002055	5.786373	5.582381
8	7.325481	7.019692	6.732745	6.463213	6.209794
9	8.162237	7.786109	7.435332	7.107822	6.801692
10	8.982585	8.530203	8.110896	7.721735	7.360087
11	9.786848	9.252624	8.760477	8.306414	7.886875
12	10.575341	9.954004	9.385074	8.863252	8.383844
13	11.348374	10.634955	9.985648	9.393573	8.852683
14	12.106249	11.296073	10.563123	9.898641	9.294984
15	12.849264	11.937935	11.118387	10.379658	9.712249
16	13.577709	12.561102	11.652296	10.837770	10.105895
17	14.291872	13.166118	12.165669	11.274066	10.477260
18	14.992031	13.753513	12.659297	11.689587	10.827603
19	15.678462	14.323799	13.133939	12.085321	11.158116
20	16.351433	14.877475	13.590326	12.462210	11.469921
21	17.012269	15.415024	14.029160	12.821153	11.764077
22	17.658048	15.936917	14.451115	13.163003	12.041582
23	18.292204	16.443608	14.856842	13.488574	12.303379
24	18.913926	16.935542	15.246963	13.798642	12.550358
25	19.523456	17.413148	15.622080	14.093945	12.783356

To find the amount of an annuity.

1. What is the amount of an annuity of \$500, at 5% compound interest, which has remained unpaid for 5 years?

$\frac{\$1.05^5-1}{.05} \times 500 = \$2762.815$, the amount required.

ANALYSIS.—The amounts of the 5 payments form a series in geometrical progression, of which \$500 is the first term, the amount of \$1 for one year (\$1.05) is the ratio, and 5, the number of years, is the number of terms. The amount of the annuity, therefore, is the sum of the series.

By using the tables showing the amount of an annuity of \$1, the work is very much diminished: thus, the amount of \$1 at 5% for 5 years = \$5.525631, which being multiplied by 500 gives \$2762.815 as the value required.

RULE.

Multiply the amount of \$1 for the given time, as found in the table, by the number denoting the annuity, and the product will be the required amount.

2. What is the value of an annuity of \$1200 which has remained unpaid for 10 years, at 5%?

3. \$120 a year invested in a loan association which earns 6% compound interest will amount to how much in 9 years?

4. What is the amount of an annuity of \$750, at 3%, which has remained unpaid for 9 years?

5. If a man deposits \$50 a year in a bank that pays 5% compound interest, how much will he save in 20 years?

6. If an annuity of \$125 remains unpaid for 10 years, what sum will discharge the debt with compound interest at 6%?

To find the present worth of an annuity.

1. What is the present worth of an annuity of $500, which is to continue for 5 years, interest compounded at 5%?

$4.329477 × 500 = $2164.7385, the present worth required.

ANALYSIS.—We find in the table, the present worth of an annuity of $1 for five years, and multiply by the number denoting the value of the given annuity (500), thus obtaining $2164.7385 as the required present worth.

RULE.

Multiply the present worth of an annuity of $1 for the given time, as found in the table, by the number denoting the annuity, and the product will be the required present worth.

2. What is the present worth of an annuity of $150, for 10 years, at 6% compound interest?

3. How much should be paid for an annuity of $50 a year, for 20 years, at 5% compound interest?

4. I wish to secure the payment of $100 a year for 15 years; what sum must I pay, if interest is compounded at 4%?

To find the value of a perpetuity.

1. What is the value of a ground-rent of $60, interest at 6%?

$60 ÷ .06 = $1000, the value required.

ANALYSIS.—To produce $.06 requires $1, and to produce $60 *will require* as many dollars as .06 is contained times in 60, or *$1000.*

RULE.

Divide the given annuity by the number denoting the rate of interest, expressed decimally.

2. What is the value of a perpetual income of $1000 a year, interest at 9%?

3. Find the value of a ground-rent of $300, one-half of which is to be paid every six months, interest at 8%.

4. What sum will extinguish a ground-rent of $20 a year, interest at 10%?

To find the annuity, the present worth, or the amount, the time, and the rate being given.

1. The present worth of an annuity, to continue for 5 years, at 5% compound interest, is $2164.7385; what is the annuity?

$2164.7385 ÷ 4.329477 = $500, the required annuity.

ANALYSIS.—If the present worth of an annuity of $1 at the given rate is $4.329477, the required annuity will be as many dollars as 4.329477 is contained times in $2164.7385, or $500.

RULE.

Divide the given present worth by the present worth of $1, or the given amount by the amount of $1, for the given time and rate, and the quotient will be the required annuity.

2. The amount of a ground-rent, which has been forborne for 5 years, is $2762.815; what is the ground-rent, interest being at 5%?

3. The present value of an annuity to be continued for 15 years, at 6% compound interest, is $9712.249; what is the annuity?

To find the value of an annuity in reversion.

1. What is the present worth of an annuity of $60, to commence in 4 years, and to continue 4 years, compound interest, at 4%?

ANALYSIS.—The present worth of an annuity of $1, payable for 8 years, at 4%, is $6.732745
The present worth of $1 for 4 years is 3.629895
The difference of these values is $3.102850
$3.102850 × 60 = $186.171, the value of the reversion.

RULE.

Find the present worth of an annuity of $1 for the full time, also for the time during which payment is deferred; the difference of these present worths multiplied by the number which denotes the annuity will be the value of the reversion.

2. I have given the reversion of the rent of a house, worth $200 a year, to my son, to commence after my wife has had the use of it for 10 years, and to continue for 10 years. What is the present value of my son's reversion?

3. What is the value of a ground-rent of $500 per annum, to commence after 20 years?

BUILDING ASSOCIATIONS.

Building Associations are organized for the purpose of accumulating a fund to be loaned to the members, on approved security. The stockholders are thus enabled to purchase real estate or other property, and to invest their savings safely and speedily.

The shares are estimated at a fixed sum, usually $200 *each, and* are paid for in monthly installments of $1 each.

The money of the association is loaned to the stockholder who offers the highest premium for its use. The premium in some instances is deducted at once, in others it is paid in monthly installments.

Interest is sometimes charged on the amount loaned, in other cases on the amount loaned plus the premium.

When the value of each share amounts to $200, the stockholders receive that sum per share, either in cash, or by the return of their own securities to that amount.

TABLE.

The amount of $1 compounded monthly, at 6% per annum, also the amount of a monthly payment of $1, compounded in the same manner.

MO.	AMOUNT OF $1.	MONTHLY PAYMENTS OF $1.	MO.	AMOUNT OF $1.	MONTHLY PAYMENTS OF $1.
1	$1.00500	$1.000	*36*	$1.19668	$39.336
2	1.01002	2.005	*48*	1.27049	54.098
3	1.01507	3.015	*60*	1.34885	69.771
4	1.02015	4.030	*72*	1.43205	86.410
5	1.02525	5.050	*84*	1.52038	104.076
6	1.03037	6.075	*90*	1.56656	113.313
7	1.03553	7.106	*96*	1.61415	122.831
8	1.04070	8.141	*102*	1.66319	132.638
9	1.04591	9.182	*108*	1.71371	142.743
10	1.05114	10.228	*114*	1.76577	153.155
11	1.05640	11.279	*120*	1.81941	163.883
12	1.06167	12.335	*126*	1.87468	174.937
18	1.09393	18.786	*132*	1.93163	186.327
24	1.12716	25.432	*138*	1.99031	198.063
30	1.16140	32.280	*144*	2.05073	210.148

To estimate the probable duration of a series of Stock, the value of a share being $200.

1. A building association has loaned its money on 500 shares of stock, at an average premium of 25%, thus paying $150 on each share, and receiving interest monthly at 6% per annum, along with the monthly installments. If there are 1000 shares of stock in all, how long will the association run before its shares are worth $200 each?

ANALYSIS.—The borrowing stockholders have received their money, and the owners of the remaining 500 shares are entitled to all the proceeds of the association. When the stock, therefore, is worth $175 a share, there will be enough to pay 500 shares at the rate of $200 each, as there will then be a margin of $25 on each share of the 500 shares that have already been paid, at $150 each. In the above table, $174.937 corresponds to 126 months; the shares will then be worth $200 each, and the society will therefore probably close in 126 months, or $10\frac{1}{2}$ years.

RULE.

Make the number of shares on which money has been loaned the numerator of a fraction, and the total number of shares, the denominator. Multiply this fraction by twice the number which denotes the average premium, and subtract the product from the final value of a share ($200). Find the time in which a monthly payment of $1 will amount to the sum thus obtained, and the number of months most nearly corresponding will be the probable duration of the series.

Note.—In the practical working of building associations, so many contingencies arise, that the value of the stock can only be determined by balancing the books.

2. What is the probable duration of an association in which the money is loaned at an average premium of 20% on $\frac{3}{4}$ of 1200 shares; the average amount loaned on a *single share* being $160?

3. A borrows $1800 for 10 years at 6% simple interest. B borrows $2000 of a building association at 10% premium. How much more does A's loan cost than B's, the association closing in 10 years?

ANALYSIS.—$1800 at 6% for 10 years amounts to $2880. B pays $20 monthly for 10 years = $2400. $2880 − $2400 = $480, additional cost of A's loan.

4. The money of a building association has been loaned at an average premium of $33\frac{1}{3}\%$ on the whole number of shares. In what number of months should its shares reach their final value of $200 each?

5. What is the probable duration of an association, whose money has been loaned at an average premium of 40% on 1100 shares, the whole number of shares being 1200?

6. If a building association closes in 8 years, having disposed of all its shares at a premium, what was the average rate obtained for the money?

CIRCULATING DECIMALS.

A Circulating Decimal is a decimal in which certain figures are continually repeated in the same order. Thus, .111+, which is the result obtained in changing $\frac{1}{9}$ to a decimal, is a circulating decimal; for if the division be continued, the figures will be found to repeat in the same order continually.

Strictly speaking, these fractions are not decimals, but *nonary fractions*, as they have unexpressed denominators whose value is always *nine*, or some multiple of nine.

A consideration of the manner in which circulating *decimals originate* will lead to the proper method of

treating them. Let it be required to change $\frac{1}{9}$ to an equivalent decimal. Here we observe that the figure 1 will be repeated continually, with a remainder, 1, which is evidently $\frac{1}{9}$ of the value of a decimal unit in the column to which it is annexed. The 1 which takes the place of this $\frac{1}{9}$, in case the division is carried further, is $\frac{1}{9}$ of the preceding decimal unit, and the entire expression might properly be written $\frac{1}{9}\frac{1}{9}\frac{1}{9} = \frac{1}{9}$.

$$\begin{array}{r|l} 9)1.00(.11\frac{1}{9} \\ \underline{9} \\ 10 \\ \underline{9} \\ 1 \end{array}$$

A **Repetend** is the figure or series of figures repeated in a circulating decimal.

The repetend is written once, and a dot (·) is placed over a single figure when there is but one, and over the first figure and the last figure of a series. Thus, $.\dot{1}$ and $.\dot{1}42578\dot{}$ are repetends.

A **Pure Circulating Decimal** contains no decimal figures but those of the repetend; as $.\dot{3}$, $.\dot{6}\dot{6}$.

A **Mixed Circulating Decimal** contains one or more decimal figures which do not form part of the repetend; as $.6\dot{3}\dot{3}\dot{3}$, $.0\dot{6}$.

The figures which precede the repetend are called the *finite part.*

A pure circulating decimal is equivalent to a fraction whose numerator is the repetend, and whose denominator is as many places of nines as there are repeating figures. For we have seen that $.\dot{1} = \frac{1}{9}$; hence $.\dot{2} = \frac{2}{9}$, $.\dot{9} = \frac{9}{9}$, etc.

A mixed circulating decimal is equivalent to a decimal *and a fraction.* Thus, $.0\dot{6} = \frac{0\frac{6}{9}}{10} = \frac{6}{9} \div 10 = \frac{6}{9} \times \frac{1}{10} = \frac{1}{15}$.

REDUCTION.

1. Change $.\dot{3}\dot{3}$ to an equivalent common fraction.

$$.\dot{3}\dot{3} = \frac{333}{999} = \tfrac{1}{3}$$

ANALYSIS.—We write the repetend without the decimal point, as the numerator, and for the denominator we take as many nines as there are places of figures in the repetend, obtaining $\frac{333}{999}$, which we reduce to its lowest terms $\frac{1}{3}$.

2. Change $.00\dot{3}$ to an equivalent common fraction.

$$.00\dot{3} = .00\tfrac{3}{9} = \frac{00\frac{3}{9}}{100} = \tfrac{3}{9} \div 100 = \tfrac{3}{9} \times \tfrac{1}{100} = \tfrac{1}{300}.$$

ANALYSIS.—We write the mixed circulating decimal as a mixed decimal $.00\frac{3}{9}$; then, omitting the decimal point, and writing the denominator 100, we have an expression of division, $\frac{3}{9} \div 100$, which we find to be equal to $\frac{1}{300}$.

RULE.

If the circulating decimal is pure, omit the decimal point, make the repetend the numerator, and take as many nines for the denominator as there are figures in the repetend.

If the circulating decimal is mixed, change it to an equivalent mixed decimal, and simplify the resulting fraction.

3. Change $.\dot{0}\dot{9}$ to an equivalent common fraction.

4. What common fraction is equivalent to $.\dot{0}76923$?

5. Reduce $.1\dot{3}\dot{3}$ to a common fraction.

6. Change $.\dot{4}28571\dot{4}$ to a common fraction.

7. What common fraction is equivalent to $.4\dot{6}\dot{6}$?

8. Reduce $.35\dot{3}$ to a common fraction.

9. Reduce $.22\dot{2}$ to a common fraction.

10. What common fraction is equivalent to $.\dot{4}2857\dot{1}\dot{4}\dot{2}$?

OPERATIONS IN THE FUNDAMENTAL RULES.

Addition and Subtraction of circulating decimals can be performed after making the repetends similar and terminating them at the same point. In Multiplication and Division of these fractions it is necessary to reduce them to common fractions and perform the required operations, afterwards changing the results to equivalent circulating decimals. The latter method is here adopted for all operations in the fundamental rules, involving repetends.

1. Find the sum of $.0\dot{6}$ and $.0\dot{3}$, and multiply the result by $.\dot{1}$.

$.0\dot{6} = \frac{0\frac{6}{9}}{10} = \frac{6}{9} \div 10 = \frac{1}{15}$; $.0\dot{3} = \frac{0\frac{3}{9}}{10} = \frac{3}{9} \div 10 = \frac{1}{30}$; $.\dot{1} = \frac{1}{9}$.

$(\frac{1}{15} + \frac{1}{30}) \times \frac{1}{9} = \frac{3}{30} \times \frac{1}{9} = \frac{1}{90}$; $\frac{1}{90} = .0\dot{1}$, product.

ANALYSIS.—We reduce $.0\dot{6}$, $.0\dot{3}$, and $.\dot{1}$ to equivalent common fractions, $\frac{1}{15}$, $\frac{1}{30}$, and $\frac{1}{9}$, and perform the required operations, afterwards changing the result to a circulating decimal.

RULE.

Change circulating decimals to common fractions, perform the required operations, and reduce the result to an equivalent decimal.

2. What is the value of $1.\dot{3} + 16.\dot{1} + 27.0\dot{6} + 13.3\dot{3}$?

3. What is the value of $(7.\dot{2} - 3.\dot{6}) \times 6.0\dot{3}$?

4. Multiply 3.5 by $(4.\dot{2} - 3.\dot{8})$.

5. From $5.\dot{8}$ subtract $\frac{1}{3}$, and divide the remainder by $(2.\dot{1} - .0\dot{3})$.

6. What is the product of $17.32\dot{5}\dot{1}$ by $.6\dot{6}$?

7. Divide $25.\dot{4}$ by $16.\dot{3}$, and from the quotient subtract $.\dot{7}$.

8. What is the value of $.3\dot{3}$ of a pound sterling?

9. Reduce $.\dot{4}28571\dot{4}$ of a week to days.

CURRENCIES.

The **Currency** of a country is its money.

The following table gives the value in U. S. money (gold) of the current coin of several of the most important countries with which the United States has commercial relations.

TABLE OF FOREIGN CURRENCIES.

COUNTRY.	COIN.	VALUE.	COUNTRY.	COIN.	VALUE.
Belgium,	Franc	$.193	German Empire,	Mark	$.238
Brazil,	Milreis	.545	Italy,	Lira	.193
Canada,	Dollar	1.000	Japan,	Yen	.997
Chili,	Peso	.912	Norway,	Crown	.268
Cuba,	"	.926	Portugal,	Milreis	1.084
Denmark,	Crown	.268	Spain,	Peseta	.193
Egypt,	Pound	4.974	Sweden,	Crown	.268
France,	Franc	.193	Switzerland,	Franc	.193
Great Britain,	Pound	4.866½	Turkey,	Piaster	.044

Note.—To change foreign currency to U. S. money, or U. S. money to foreign currency, we multiply or divide, as the case may require, by the value in U. S. money of a unit of the given currency.

1. Change 500 Spanish pesetas to dollars and cents.

2. Change $100 to £ s. d. English money.

3. What is the value of £100 in dollars and cents?

4. A Frenchman visiting the National Exposition brought with him 2000 francs in gold; what was it worth in U. S. money?

5. What is the value of 1000 francs in £ s. d.?

6. What is the value of $150 in Danish crowns?

7. Change 1500 marks of Germany to francs.

8. A bill for port wine at the vineyard amounted to 600 milreis; how many dollars and cents were required to *pay it?*

EXCHANGE..

Exchange is the receiving or paying of money in one place for its value in any other place, by means of drafts or bills of exchange.

A **Bill of Exchange** is a written order for the payment of money, drawn in one place and made payable in another.

When the exchange is between different places in the same country, it is called **Inland** or **Domestic Exchange,** and the domestic bill of exchange is usually called a **Draft.**

When the exchange is between different countries, it is called **Foreign Exchange.**

A **Sight Bill** is one which is payable on its presentation.

A **Time Bill** is one which is payable in a specified time after its date, or after presentation.

Note 1.—Three days of grace are allowed on *time drafts.*

Note 2.—The *maker* or *drawer* of a bill is the person who orders the money to be paid; the *drawee* is the one on whom it is drawn, the *payee* is the one to whom it is to be paid, and the *remitter* is the one who sends it.

An **Acceptance** is an agreement on the part of the drawee to pay the bill in accordance with the conditions named therein, and is indicated by his writing the word "accepted," together with his name and the date, across the face of the bill.

INLAND OR DOMESTIC EXCHANGE.

CASE I.

To find the cost of a draft.

1. What is the cost of a sight draft on New Orleans for *$10000, at* $\frac{1}{8}$ of 1 % premium?

ANALYSIS.—$1 + $.00125 = $1.00125, cost of $1. $1.00125 × 10000 = $10012.50, cost of draft.

2. What is the cost of a draft on St. Louis for $3500, at 60 days, the premium being 1¼%, interest 6%?

ANALYSIS.—$1.0125 = cost of $1 at sight. $.0105 = bank discount on $1 for 63 days. $1.0125 − $.0105 = $1.002, cost of $1 at 63 days. $1.002 × 3500 = $3507, cost of draft.

RULE.

Multiply the cost of $1 by the amount of the draft.

3. What is the cost of a sight draft on San Francisco for $1500, at 1⅝% premium?

4. What is the cost of a draft on Chicago for $800, at 30 days, the discount being 1%, interest 7%?

5. Find the cost of the following draft at ⅜% premium, interest at 6%.

Philadelphia, January 1, 1877.

$500.

Sixty days after date, pay to the order of Edward Walton, five hundred dollars, value received, and charge the same to my account.

WILLIAM JOHNSON.

To JAMES MACINTOSH & Co., New York.

CASE II.

To find the face of a draft.

1. Bought a draft in Kansas City on Philadelphia, payable in 60 days, for $2536.25; what was the face of the draft, exchange being 2½% premium, interest 6%?

ANALYSIS.—$1.025 = cost of $1 at sight. $.0105 = bank discount on $1 for 63 days. $1.025 − $.0105 = $1.0145, cost of $1 at 63 days. $2536.25 ÷ 1.0145 = $2500, face of draft.

RULE.

Divide the given cost by the cost of $1.

2. What is the face of a sight draft, purchased for $690.20, premium at $1\frac{1}{2}\%$?

3. What is the face of a sight draft, purchased for $1449.05, discount at $\frac{3}{4}\%$?

4. Bought a draft on Chicago for $1430.65, payable in 90 days, discount 1%, interest 7%; what was the face?

5. Bought a draft on Philadelphia for $2049, payable in 60 days, premium $3\frac{1}{2}\%$, interest 6%; what was the face?

FOREIGN EXCHANGE.

Foreign Bills of Exchange are drawn in sets, consisting of three bills of the same tenor and date, called the *First*, *Second*, and *Third* of exchange, respectively. These are sent by different mails, in order to save time in case of loss or miscarriage, and when either of the three has been paid the others are void.

Sterling Bills are bills of exchange drawn on England, Ireland, or Scotland. The rate of exchange, commercially called the *par* of exchange, between the United States and Great Britain, has been fixed at $4.8665 for the value of £1 sterling.

CASE I.

To find the cost of a bill on England.

1. What is the cost in currency of a bill of exchange on London for £200 10 s., exchange at par, and gold at 106?

Analysis.—£200 10 s. = £200.5.

$4.8665 × 200.5 = $975.73, cost in gold.

$975.73 × 1.06 = $1034.27, cost in currency.

RULE.

Multiply the market value of £1 by the amount named in the bill.

2. What is the cost in gold of a bill on Glasgow for £1760 16 s. 6 d., exchange at $4.875?

3. £290 2 s. 6d. *Baltimore, Jan. 5, 1877.*

At sight of this *first* of exchange (second and third of same tenor and date unpaid) pay to the order of Henry Preston, two hundred and ninety pounds, two shillings, and six pence, value received, and charge the same to

HIRAM AYRES & Co.

To SMITH BROS. & Co., London.

Find the cost of the above bill in currency, exchange at $4.87, gold 109½.

CASE II.

To find the face of a bill on England.

1. I have $1039.91 to remit to London; for what number of pounds sterling must a bill be drawn, exchange at $4.87, gold 106½?

ANALYSIS.—$1039.91 ÷ 1.06½ = $976.44, value in gold.

976.44 ÷ 4.87 = 200.5.

£200.5 = £200 10 s., the amount for which the bill must be drawn.

RULE.

Divide the value in gold of the given amount by the market value of £1.

2. A merchant in Louisville gave $1000 in gold for a bill on London, exchange at $4.86½; what was the face of the bill?

3. A bill on Liverpool cost $3000 in currency; what number of pounds sterling did it call for, exchange at $4.86, gold 108½?

Exchange on other countries is calculated in a similar manner to exchange on England, substituting the value of the coin used as the basis of exchange, in place of the value of a pound sterling.

4. What is the cost in gold of a bill on Geneva for 2500 francs, exchange at 5.17 francs to the dollar?

THE METRIC SYSTEM.

The **Metric System** is a decimal system of weights and measures, and derives its name from the **meter** which is the unit of the system.

Note.—The metric system originated in France, and was finally made obligatory in that country in 1841. In 1866 its use in the United States was authorized by Act of Congress.

The length of the meter was intended to be one ten-millionth of the distance from the equator to either pole, measured at the level of the sea; but it is in reality a trifle less.

The higher denominations of any measure, obtained by multiplication of the unit, are named by prefixing to the name of the unit of that measure, the *Greek* numerals Deka, 10, Hecto, 100, Kilo, 1000, or Myria, 10000.

The lower denominations of any measure, obtained by division of the unit, are named by prefixing to the name *of the unit of* that measure, the *Latin* numerals Deci, $\frac{1}{10}$, *Centi*, $\frac{1}{100}$, or Milli, $\frac{1}{1000}$.

ABBREVIATIONS.

The name of each leading unit is abbreviated by writing its first letter after the number denoting the given quantity; 1 l. = 1 liter, 5 m. = 5 meters, etc. The Greek names of the higher orders of units are abbreviated in capitals; Hg. = Hektogram, Kg. = Kilogram, etc. The Latin names of the lower orders of units are abbreviated in small letters; ds. = decistere, mm. = millimeter, etc. In square measure and cubic measure, sq. and cu. = square and cubic respectively; 5 sq. Dm. = 5 square Dekameters; 9 cu. dm. = 9 cubic decimeters.

MEASURES OF LENGTH.

The unit of length is the **Meter.**

10 Millimeters = 1 Centimeter = .3937079 inches.
10 Centimeters = 1 Decimeter = 3.937079 "
10 Decimeters = 1 **METER** = 39.37079 "
10 Meters = 1 Dekameter = 393.7079 "
10 Dekameters = 1 Hektometer = 3937.079 "
10 Hektometers = 1 Kilometer = 39370.79 "
10 Kilometers = 1 Myriameter = 393707.9 "

Note.—The Meter is used in the same manner as our yard, and the Kilometer as our mile.

MEASURES OF SURFACE.

The unit for measuring ordinary surfaces is the **Square Meter**; for measuring land, the **Are** or square dekameter.

100 Sq. Millimeters = 1 Sq. Centimeter = .155 sq. in.
100 Sq. Centimeters = 1 Sq. Decimeter = 15.5 "
100 Sq. Decimeters = 1 **SQ. METER** = 1550 "

100 Centiares (sq. meters) = 1 **ARE** = 119.6 sq. yd.
100 Ares = 1 Hektare = 2.471 acres.

MEASURES OF CAPACITY.

The unit of wood measure is the **Stere;** it is equal to the **Cubic Meter,** the unit for other measurements of solids.

1000 Cu. Millimeters = 1 Cu. Centimeter = .061 cu. in.
1000 Cu. Centimeters = 1 Cu. Decimeter = 61.027 "
1000 Cu. Decimeters = 1 **CU. METER** = 35.316 cu. ft.

10 Decisteres = 1 **STERE** (cu. meter) = 35.316 cu. ft.
10 Steres = 1 Dekastere = 13.080 cu. yd.

The unit of dry measure and of liquid measure is the **Liter,** which is equal to a cubic decimeter.

		DRY MEASURE.	LIQUID MEASURE.
10 Milliliters = 1	Centiliter	= 0.6102 cu. in.	or 0.338 fl. oz.
10 Centiliters = 1	Deciliter	= 6.1022 "	" 0.845 gi.
10 Deciliters = 1	**LITER**	= 0.908 qt.	" 1.0567 qt.
10 Liters = 1	Dekaliter	= 9.08 "	" 2.6417 gal.
10 Dekaliters = 1	Hektoliter	= 2.8375 bu.	" 26.417 "
10 Hektoliters = 1	Kiloliter	= 28.375 "	" 264.17 "
10 Kiloliters = 1	Myrialiter	= 283.75 "	" 2641.7 "

MEASURES OF WEIGHT.

The unit of weight is the **Gram.**

10 Milligrams = 1 Centigram = 0.1543 grains.
10 Centigrams = 1 Decigram = 1.5432 "
10 Decigrams = 1 **GRAM** = 15.432 "
10 Grams = 1 Dekagram = 0.3527 oz. Avoir.
10 Dekagrams = 1 Hektogram = 3.527 " "
10 Hektograms = 1 Kilogram = 2.2046 lb. "
10 Kilograms = 1 Myriagram = 22.046 " "
10 Myriagrams = 1 Quintal = 220.46 " "
10 Quintals = 1 Tonneau = 2204.6 " "

Value of Denominate Numbers in Terms of the Metric System.

LONG MEASURE.

INCHES.

1 Inch	= 2.540 Centimeters.	7 Inches	= 1.778 Decimeters.
2 "	= 5.080 "	8 "	= 2.032 "
3 "	= 7.620 "	9 "	= 2.286 "
4 "	= 1.016 Decimeters.	10 "	= 2.540 "
5 "	= 1.270 "	11 "	= 2.794 "
6 "	= 1.524 "	12 "	= 3.048 "

FEET.

2 Feet	= 6.096 Decimeters.	3 Feet	= 9.144 Decimeters.

YARDS.

1 Yard	= 9.144 Decimeters.	4 Yards	= 3.657 Meters.
2 "	= 1.828 Meters.	5 "	= 4.572 "
3 "	= 2.743 "	5½ "	= 5.029 "

RODS.

1 Rod	= 5.0291 Meters.	6 Rods	= 3.0174 Dekameters.
2 "	= 1.0058 Dekameters.	7 "	= 3.5203 "
3 "	= 1.5087 "	8 "	= 4.0232 "
4 "	= 2.0116 "	9 "	= 4.5261 "
5 "	= 2.5145 "	10 "	= 5.0291 "

MILES.

1 Mile	= 1.6093 Kilometers.	6 Miles	= 9.6558 Kilometers.
2 "	= 3.2186 "	7 "	= 11.2651 "
3 "	= 4.8279 "	8 "	= 12.8744 "
4 "	= 6.4372 "	9 "	= 14.4837 "
5 "	= 8.0465 "	10 "	= 16.0931 "

Note.—As the denominations in the Metric System change their names in accordance with the decimal system, the same quantity may be readily expressed in terms of any of its denominations by a mere change of the decimal point. Thus, 1.6 Kilometers may be written .16 Myriameter, or as 16 Hektometers, 160 Dekameters, etc.; it may also be read as 1 Kilometer 6 Hektometers.

SQUARE MEASURE.

SQUARE INCHES.

1 Sq. In.	= 6.4528	Sq. Centimeters.		6 Sq. In.	= 38.7168	Sq. Centimeters.	
2 "	= 12.9056	"		7 "	= 45.1696	"	
3 "	= 19.3584	"		8 "	= 51.6224	"	
4 "	= 25.8112	"		9 "	= 58.0752	"	
5 "	= 32.2640	"		10 "	= 64.5280	"	

SQUARE FEET.

1 Sq. Ft.	= 9.29	Sq. Decimeters.		6 Sq. Ft.	= 55.74	Sq. Decimeters.	
2 "	= 18.58	"		7 "	= 65.03	"	
3 "	= 27.87	"		8 "	= 74.32	"	
4 "	= 37.16	"		9 "	= 83.61	"	
5 "	= 46.45	"		10 "	= 92.90	"	

SQUARE YARDS.

1 Sq. Yd.	= .8361	Sq. Meters.		6 Sq. Yd.	= 5.0166	Sq. Meters.	
2 "	= 1.6722	"		7 "	= 5.8527	"	
3 "	= 2.5083	"		8 "	= 6.6888	"	
4 "	= 3.3444	"		9 "	= 7.5249	"	
5 "	= 4.1805	"		10 "	= 8.3610	"	

SQUARE RODS.

1 Sq. Rd.	= 25.29	Centiares.		6 Sq. Rd.	= 1.5175	Ares.	
2 "	= 50.58	"		7 "	= 1.7704	"	
3 "	= 75.87	"		8 "	= 2.0233	"	
4 "	= 1.0117	Ares.		9 "	= 2.2763	"	
5 "	= 1.2646	"		10 "	= 2.5292	"	

ACRES.

1 A.	= 40.467	Ares.		6 A.	= 2.4280	Hektares.	
2 "	= 80.934	"		7 "	= 2.8327	"	
3 "	= 1.2140	Hektares.		8 "	= 3.2374	"	
4 "	= 1.6187	"		9 "	= 3.6420	"	
5 "	= 2.0233	"		10 "	= 4.0467	"	

CUBIC MEASURE.

CUBIC INCHES.

1 Cu. In.	= 16.39 Cu. Centimeters.	6 Cu. In.	= 98.32 Cu. Centimeters.
2 "	= 32.77 "	7 "	= 114.70 "
3 "	= 49.16 "	8 "	= 131.09 "
4 "	= 65.54 "	9 "	= 147.47 "
5 "	= 81.93 "	10 "	= 163.86 "

CUBIC FEET.

1 Cu. Ft.	= 28.32 Cu. Decimeters.	6 Cu. Ft.	= 169.89 Cu. Decimeters.
2 "	= 56.63 "	7 "	= 198.20 "
3 "	= 84.95 "	8 "	= 226.52 "
4 "	= 113.26 "	9 "	= 254.84 "
5 "	= 141.58 "	10 "	= 283.15 "

CUBIC YARDS.

1 Cu. Yd.	= .7645 Cu. Meters.	6 Cu. Yd.	= 4.5870 Cu. Meters.
2 "	= 1.5290 "	7 "	= 5.3515 "
3 "	= 2.2935 "	8 "	= 6.1160 "
4 "	= 3.0580 "	9 "	= 6.8806 "
5 "	= 3.8225 "	10 "	= 7.6451 "

LIQUID MEASURE.

GILLS.

1 Gill	= 1.1831 Deciliters.	3 Gills	= 3.5493 Deciliters.
2 "	= 2.3662 "	4 "	= 4.7325 "

PINTS.

1 Pint	= 4.7325 Deciliters.	2 Pints	= 9.4650 Deciliters.

QUARTS.

1 Quart	= 9.4650 Deciliters.	3 Quarts	= 2.8395 Liters.
2 "	= 1.8930 Liters.	4 "	= 3.7860 "

GALLONS.

1 Gallon	= 3.786 Liters.	6 Gallons	= 2.271 Dekaliters.
2 "	= 7.572 "	7 "	= 2.650 "
3 "	= 1.135 Dekaliters.	8 "	= 3.028 "
4 "	= 1.514 "	9 "	= 3.407 "
5 "	= 1.893 "	10 "	= 3.786 "

DRY MEASURE.

PINTS.

1 Pint = 5.5067 Deciliters. 2 Pints = 1.1013 Liters.

QUARTS.

1 Quart	= 1.1013 Liters.	5 Quarts	= 5.5067 Liters.
2 "	= 2.2027 "	6 "	= 6.6081 "
3 "	= 3.3040 "	7 "	= 7.7094 "
4 "	= 4.4054 "	8 "	= 8.8108 "

PECKS.

1 Peck	= 8.8108 Liters.	3 Pecks	= 2.6432 Dekaliters.
2 "	= 1.7621 Dekaliters.	4 "	= 3.5243 "

BUSHELS.

1 Bushel	= 3.524 Dekaliters.	6 Bushels	= 2.114 Hektoliters.
2 "	= 7.048 "	7 "	= 2.467 "
3 "	= 1.057 Hektoliters.	8 "	= 2.819 "
4 "	= 1.409 "	9 "	= 3.171 "
5 "	= 1.762 "	10 "	= 3.524 "

AVOIRDUPOIS WEIGHT.

OUNCES.

1 Ounce	= 2.835 Dekagrams.	9 Ounces	= 2.551 Hektograms.
2 "	= 5.671 "	10 "	= 2.835 "
3 "	= 8.506 "	11 "	= 3.119 "
4 "	= 1.134 Hektograms.	12 "	= 3.402 "
5 "	= 1.417 "	13 "	= 3.685 "
6 "	= 1.701 "	14 "	= 3.969 "
7 "	= 1.984 "	15 "	= 4.252 "
8 "	= 2.268 "	16 "	= 4.536 "

POUNDS.

1 Pound	= 4.5359 Hektograms.	6 Pounds	= 2.7216 Kilograms.
2 "	= 9.0718 "	7 "	= 3.1751 "
3 "	= 1.3608 Kilograms.	8 "	= 3.6227 "
4 "	= 1.8144 "	9 "	= 4.0823 "
5 "	= 2.2680 "	10 "	= 4.5359 "

2000 " = .9072 Tonneau. 2240 " = 1.0160 Tonneaux.

APOTHECARIES' WEIGHT.

GRAINS.

1	Grain	= 6.480	Centigrams.	11	Grains	= 7.1280	Decigrams.
2	"	= 1.296	Decigrams.	12	"	= 7.7760	"
3	"	= 1.944	"	13	"	= 8.4240	"
4	"	= 2.592	"	14	"	= 9.0720	"
5	"	= 3.240	"	15	"	= 9.7200	"
6	"	= 3.888	"	16	"	= 1.0368	Grams.
7	"	= 4.536	"	17	"	= 1.1016	"
8	"	= 5.184	"	18	"	= 1.1664	"
9	"	= 5.832	"	19	"	= 1.2312	"
10	"	= 6.480	"	20	"	= 1.2960	"

SCRUPLES.

2	Scruples	= 2.5920	Grams.
3	Scruples	= 3.8879	Grams.

DRACHMS.

1	Drachm	= 3.8879	Grams.	5	Drachms	= 1.9440	Dekagrams.
2	"	= 7.7758	"	6	"	= 2.3328	"
3	"	= 1.1664	Dekagrams.	7	"	= 2.7216	"
4	"	= 1.5552	"	8	"	= 3.1103	"

OUNCES.

1	Ounce	= 3.1103	Dekagrams.	7	Ounces	= 2.1772	Hektograms.
2	"	= 6.2206	"	8	"	= 2.4883	"
3	"	= 9.3309	"	9	"	= 2.7993	"
4	"	= 1.2441	Hektograms.	10	"	= 3.1103	"
5	"	= 1.5552	"	11	"	= 3.4214	"
6	"	= 1.8662	"	12	"	= 3.7324	"

POUNDS.

1	Pound	= 3.7324	Hektograms.	6	Pounds	= 2.2394	Kilograms.
2	"	= 7.4648	"	7	"	= 2.6127	"
3	"	= 1.1197	Kilograms.	8	"	= 2.9859	"
4	"	= 1.4929	"	9	"	= 3.3592	"
5	"	= 1.8662	"	10	"	= 3.7324	"

Note.—*The Troy pound, ounce, and grain have the same values as in Apothecaries' Weight.*

APOTHECARIES' LIQUID MEASURE.

MINIMS.

1 Minim	= .0616 Milliliters.	6 Minims	= .3697 Milliliters.
2 "	= .1232 "	7 "	= .4313 "
3 "	= .1848 "	8 "	= .4930 "
4 "	= .2465 "	9 "	= .5546 "
5 "	= .3081 "	10 "	= .6162 "

FLUID DRACHMS.

1 Fl. Dr.	= 3.6973 Milliliters.	5 Fl. Dr.	= 1.8486 Centiliters.
2 "	= 7.3946 "	6 "	= 2.2184 "
3 "	= 1.1092 Centiliters.	7 "	= 2.5881 "
4 "	= 1.4789 "	8 "	= 2.9578 "

FLUID OUNCES.

1 Fl. Oz.	= 2.9578 Centiliters.	9 Fl. Oz.	= 2.6620 Deciliters.
2 "	= 5.9156 "	10 "	= 2.9578 "
3 "	= 8.8734 "	11 "	= 3.2536 "
4 "	= 1.1831 Deciliters.	12 "	= 3.5494 "
5 "	= 1.4789 "	13 "	= 3.8452 "
6 "	= 1.7747 "	14 "	= 4.1409 "
7 "	= 2.0705 "	15 "	= 4.4367 "
8 "	= 2.3662 "	16 "	= 4.7325 "

PINTS.

1 Pint	= 4.7325 Deciliters.	5 Pints	= 2.3662 Liters.
2 "	= 9.4650 "	6 "	= 2.8395 "
3 "	= 1.4197 Liters.	7 "	= 3.3127 "
4 "	= 1.8930 "	8 "	= 3.7860 "

GALLONS.

1 Gallon	= 3.786 Liters.	6 Gallons	= 2.271 Dekaliters.
2 "	= 7.572 "	7 "	= 2.650 "
3 "	= 1.135 Dekaliters.	8 "	= 3.028 "
4 "	= 1.514 "	9 "	= 3.407 "
5 "	= 1.893 "	10 "	= 3.786 "

PROBLEMS.

1. What is the value of 1 ʒ 4 ʒ ı0 gr. in dekagrams?

ANALYSIS.—1 ʒ = 3.1103 dekagrams, 4 ʒ = 1.5552 dekagrams, and 10 gr. = 6.480 decigrams; removing the decimal point two places to the left, we change the decigrams to dekagrams. 3.1103 + 1.5552 + .0648 = 4.7303 dekagrams, or 4 dekagrams 7 grams 3 decigrams 3 milligrams.

2. Change 11 ʒ 6 ʒ 2 ℈ 19 gr. to hektograms.

3. Reduce 5 ʒ 1 ℈ 17 gr. to grams.

4. In 9 ʒ 7 ʒ 13 gr. how many decigrams?

5. How many centiliters in 9 f ʒ 20 ♏?

6. Change 3 Cong. 4 O. 7 f ʒ to liters.

7. How many deciliters in 3 f ʒ 50 ♏?

8. How many kilometers in 5 mi. 40 rd. 2 yd. 2 ft. 6 in.?

ANALYSIS.—5 mi. = 8.0465 kilometers, 40 rd. = 20.1164 dekameters or .201164 kilometers, 2 yd. = 1.8288 meters or .0018288 kilometers, 2 ft. = 6.096 decimeters or .0006096 kilometers, and 6 in. = 1.524 decimeters or .0001524 kilometers. 8.0465 + .201164 + .0018288 + .0006096 + .0001524 = 8.25025 + kilometers. The same result can be obtained by changing the denominate number to the decimal of a mile, and multiplying by the value of 1 mile in kilometers; thus, 5.126609 × 1.6093 = 8.25025 + kilometers.

9. Reduce 29 mi. 55 rd. 11 in. to kilometers.

10. Change 5¼ yards to centimeters.

11. Change 15 T. 12 cwt. 25 lb. 12 oz. to tonneaux.

12. In 500 lb. how many kilograms?

13. Change 625 sq. rd. to ares.

14. In 40 A. 150 rd. how many hektares?

15. How many cubic meters in 127 cubic yards?

16. Change 11 cu. yd. to steres.

17. In 35 cords 15 cord feet, how many steres?

18. *In 10* barrels 27 gallons, how many kiloliters?

19. Change 30 bushels 3 pecks to hektoliters.

20. Change 19 dekagrams to Avoirdupois ounces.

ANALYSIS.—1 dekagram = .3527 oz., and 19 dekagrams = .3527 × 19 = 6.7013 oz. Avoirdupois.

21. Reduce 17.5 kilograms to pounds Avoirdupois.

22. In 8.304 hektares how many acres?

23. In 6.543 grams how many grains?

24. Change 7.854 kiloliters to gallons.

25. In 5.32 deciliters how many fluid ounces?

26. Change 9.223 steres to cords and cord feet.

27. At 10½ cents per pound, what is the value of 1.762 myriagrams of sugar?

ANALYSIS.—1.762 × 22.046 × 10.5 = 407.8 cents, or $4.078.

28. What is the cost of 6.74 tonneaux of coal, at $5.25 per ton of 2240 pounds?

29. Bought 2.25 dekameters of cloth at $5.15 per meter, and sold it at $5.30 per yard. Did I gain or lose, and how much?

30. What is the cost of 17 cords 12 cubic feet of wood, at $2.25 per stere?

31. What price per dekameter is equal to £1 6 s. 6 d. per yard?

32. A cubical block of marble measures 3.14 meters in height; what is its weight in kilograms?

33. The diameter of a circle is 6.5 decimeters; what is its area in square inches?

34. Divide the difference between 3.14 kilometers and 175 decimeters by 2.5 centimeters.

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